COMBINED CODING, MODULATION, CARRIER, AND FRAME SYNCHRONIZATION ON THE BASIS OF TIME-VARIANT CODED 8/4-PSK

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Abstract —

In this contribution, on one hand, a frame synchronization is presented that is included in time-variant coded modulation without any loss in data rate. Furthermore, it delivers the phase offset to control the carrier loop. On the other hand, time-variant coded modulation based on block codes is defined. Hitherto, only trellis-coded schemes have been published.

1 Introduction

Since Ungerboecks trellis-coded modulation schemes [1] have been studied under practical conditions (see e.g. [2]), it has been realized that phase synchronization is far more critical, compared to the uncoded system. A coded 8-PSK scheme, of course, has a half as wide hold range of the carrier phase loop as an uncoded 4-PSK. Unfortunately, the original trellis-codes published by Ungerboeck only were 180°-invariant. This means, that if the phase difference at the comparator exceeds the hold range of $(-\pi/8, \pi/8)$, the phase loop runs through a wide random-walk zone, leading to an error burst. For this reason, some work has been done to derive 45°- or at least 90°-invariant codes. Ungerboeck [3] searched for nonlinear trellis-codes, Wei [4] published multidimensional ones and recently, Massey [5] and his group had some interesting results with convolutional codes over the ring of integers (mod 8 for 8-PSK). In [6], the author derived such 45°-phase-invariant codes based on block codes (compare also [7]). These achievements lead to charakteristics with smaller or even no random-walk zone. Long error bursts are avoided. But, of course, the width of the hold range is unchanged. Thus, the probability of cycle slips is the same, although a new stable working point is reached at once.

Furthermore, for some other reasons (e. g. coderate or complexity), it may be unfavourable to use that special phase-invariant codes. Time-variant schemes have been developed to enlarge the hold range [8], [9]. These insert 4-PSK symbols in the sequence of 8-PSK symbols. Successions of the forms 8/8/8/4, 8/8/4, 8/4/4, 8/4 etc. have been studied. Although deduced from convolutional codes, those methods require a frame synchronization to locate the 4-PSK symbols. No satisfactory proposal has been made on this aspect.

In this contribution a method for frame synchronization is described that only uses the sequence of 4-PSK subsets. There will be no need for additional sync-headers. A detailed derivation of the corresponding probabilities to be out of sync has been included to simplify the selection of sync-sequences for a special application. Furthermore, time-variant modulation schemes on the basis of block codes will be given. Zinoviev's Generalized Concatenated Codes [10], [11], [12] will prove to be very well suited to define these time-varying modulation alphabets.

2 Frame synchronisation for time-variant 8/4-PSK modulation

To improve phase synchronisation, segments of the sequence of 8-PSK symbols are restricted to be in a 4-PSK subset. Each one of the two subsets may be chosen. Thus, it would also be possible to select them according to well-known binary sync-sequences, such as m- or Barker-sequences.

As an example, the resulting succession of 4-PSK subsets for a Barker sequence of length 7 is given in Figure 1.

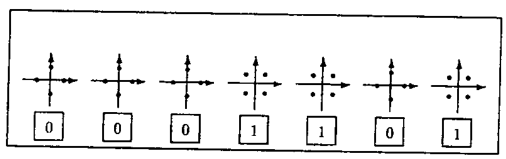


Figure 1: Barker sequence of length 7 (0,0,0,1,1,0,1), represented by the selection of 4-PSK subsets from the 8-PSK

If these signals are raised to the power of four, the original binary sequence is recovered. Described formally:

$$\begin{aligned}(e^{jn\pi/2+\varphi_0})^4 &= e^{jn2\pi}e^{j4\varphi_0} = e^{j4\varphi_0}\\ (e^{j(\pi/4+n\pi/2+\varphi_0)})^4 &= e^{j(\pi+n2\pi)}e^{j4\varphi_0} = -1\cdot e^{j4\varphi_0}\,.\end{aligned} \tag{1}$$

To combine phase and frame synchronisation, after this exponentiation, a complex cross-correlation

$$R_{AB}(s) = \sum_{i=1}^{N} A_i B^*_{s+i} \tag{2}$$

- A_i : exponentiated received samples
- B_i : components of the original binary sequence
- N : length of sync-sequence
- s : shift
- $*$: conjugated (can be omitted, because the B_i are real.)

with the original binary (±1) sequence is applied. The maximum of its absolute value determines the position of the frame synchronisation.

The phase of that maximum uniquely equals the phase offset if it is within an interval of $(-\pi, +\pi]$. This corresponds to $(-\pi/4, +\pi/4]$ for the carrier phase of the received signal before exponentiation. For example, a phase shift of $\frac{\pi}{4}$ interchanges $+1$ and -1 in the sequence to be correlated.

The proposed method for frame synchronization by selecting 4-PSK subsets from the 8-PSK maintains the desired hold range of $(-\frac{\pi}{4}, \frac{\pi}{4}]$ for the embedded 4-PSK section and also delivers the momentary phase offset to control the carrier loop. Thus, a combined frame and carrier synchronization is obtained without the need of inserting additional sync-headers, not reducing available data rate.

To select appropriate sync-sequences, probabilities to be out of sync have to be determined. In the following section upper bounds are derived, assuming additive white Gaussian noise. Other probability densities can be treated in the same way, because most of the derivation does not depend on the form of the density function.

3 The probability to be out of sync

The exact computation of the probabilities is rather complicated and not feasible, because N-fold convolutions of non-Gaussian probability density functions, resulting from the exponentiation, would be necessary. However, for selecting suitable sequences, it should be sufficient to have an upper bound of the 'sync-error probability'. This is determined assuming a discrete cross-correlation instead of the continous one given in (2). Thus, at first, a 'biterror probability' has to be derived from the Gaussian noise. The 'sync-error probability' itself can then be given according to formulas in [13].

To determine the 'biterror probability', the probability density function of the Gaussian noise in polar coordinates is needed. Assuming a two-dimensional normal density function with mean (1,0)

$$f_{xy}(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-1)^2+y^2}{2\sigma^2}}, \tag{3}$$

we obtain

$$\begin{aligned} f_{xy}(x,y)\cdot dx\cdot dy &= \frac{1}{2\pi\sigma^2} e^{-\frac{(a\cos\varphi-1)^2+a^2\sin^2\varphi}{2\sigma^2}} \cdot a\cdot da\cdot d\varphi \\ \Rightarrow\quad f_{a\varphi}(a,\varphi) &= \frac{e^{-\frac{1}{2\sigma^2}}}{2\pi\sigma^2}\, a\, e^{-\frac{(a^2-2a\cos\varphi)}{2\sigma^2}} \end{aligned} \tag{4}$$

$f_\varphi(\varphi)$ is obtained by applying

$$f_\varphi(\varphi) = \int_0^\infty f_{a\varphi}(a,\varphi)\, da\,. \tag{5}$$

With the aid of [14] (p. 338, 3.462, No. 5) or [15] (p. 167, (4.2.102), (4.2.103)) this leads to

$$f_\varphi(\varphi) = \frac{e^{-\frac{1}{2\sigma^2}}}{2\pi}\left[1+\cos\varphi\frac{\sqrt{\pi}}{\sqrt{2}\sigma}\right]\cdot e^{\frac{\cos^2\varphi}{2\sigma^2}}\left[1+\operatorname{erf}\left(\frac{\cos\varphi}{\sqrt{2}\sigma}\right)\right], \tag{6}$$

where $\operatorname{erf}(x) = 1-\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$.

Regarding Figure 2, the 'biterror probability' after exponentiation and binary hard decision is determined by

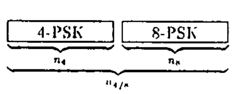
$$p = 1 - 2\cdot\int_0^{\pi/8} f_\varphi(\varphi)d\varphi - 2\cdot\int_{3\pi/8}^{5\pi/8} f_\varphi(\varphi)d\varphi - 2\cdot\int_{\pi-\pi/8}^{\pi} f_\varphi(\varphi)d\varphi. \tag{7}$$

For small standard deviation σ this can be approximated by $p \approx 1 - 2\cdot\int_0^{\pi/8} f_\varphi(\varphi)d\varphi$.

A further approximation may be given by

$$p \approx 1 - 2\cdot\int_0^{\sin\pi/8}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}dx = \operatorname{erfc}\left(\frac{\sin\pi/8}{\sqrt{2}\sigma}\right). \tag{8}$$

In the sequel, let N be the length of the binary sync-sequence and let the time-variant 8/4-PSK consist of n_4 4-PSK symbols and n_8 8-PSK symbols in a modulation interval:



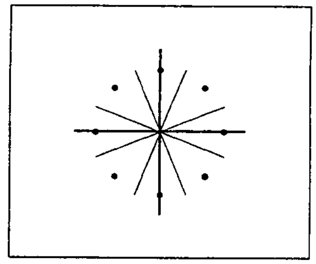


Figure 2: 8-PSK with thresholds

According to [13], the probability of having Θ negative multiplications (negative correlated bits) in a cross-correlation of length N is given by

$$\begin{aligned} P_{matr}(s,\Theta) \;=\; & \sum_{\Theta_+=T}^{R}\binom{n_+}{\Theta_+}p^{\Theta_+}(1-p)^{n_+-\Theta_+} \\ & \cdot\sum_{\Theta_-=U}^{S}\binom{n_-}{\Theta_-}(1-p)^{\Theta_-}p^{n_--\Theta_-} \\ & \cdot\binom{n_0}{\Theta-\Theta_+-\Theta_-}0.5^{n_0} \end{aligned} \tag{9}$$

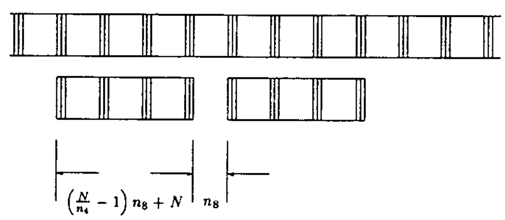
$R = \min(n_+,\Theta)$ | $S = \min(n_-,\Theta-\Theta_+)$
$T = \max(0,\Theta-n_0-n_-)$ | $U = \max(0,\Theta-n_0-\Theta_+)$

$$N = n_+ + n_- + n_0$$

$n_+(s)$:	number of positive correlated bits	(error-free)
$n_-(s)$:	number of negative correlated bits	(error-free)
$n_0(s)$:	number of uncorrelated bits	(error-free)

Θ negative correlated positions mean a cross-correlation of $R = N - 2\Theta$.

It is assumed that the sync-sequence is repeated periodically in direct succession. Then, the sync-position can be found in a certain interval of shifts s. This has to be chosen such that no overlap occurs (see the drawing below). [1]



Regarding a left shift of the righthand sequence and a right shift of the lefthand sequence, both overlap at

$$s' \stackrel{!}{=} \left(\frac{N}{n_4}-1\right)n_8 + N + n_8 - s' \Longrightarrow 2s' = \frac{N}{n_4}n_8 + N = N\left(1+\frac{n_8}{n_4}\right) \tag{10}$$

Two cases have to be distinguished:

1) If $s_{max} := \left\lfloor N\left(1+\frac{n_8}{n_4}\right)/2\right\rfloor = N\left(1+\frac{n_8}{n_4}\right)/2$:

$$\Rightarrow -s_{max}+1 \le s \le s_{max} \tag{11}$$

[1] $\lfloor\frac{N}{n_4}\rfloor = \frac{N}{n_4}$ assumed.

2) If $s_{max} := \lfloor N(1+\frac{n_8}{n_4})/2 \rfloor < N(1+\frac{n_8}{n_4})/2$:

$$\Rightarrow -s_{max} \leq s \leq s_{max} \tag{12}$$

Noting that 8-PSK symbols are to be seen as uncorrelated with the sequence (incorporated in n_0), which means positive or negative correlation appears with probability 1/2, the probability to be out of sync follows to be (see [13]):

1)

$$P_f = 1 - \sum_{\Theta_0=0}^{N-1} \left[\prod_{s=1}^{s_{max}-1} \left(\sum_{\Theta=\Theta_0+1}^{N} P_{matr}(s,\Theta) \right) \right]^2 \cdot \sum_{\Theta=\Theta_0+1}^{N} P_{matr}(s_{max},\Theta) \cdot P_{matr}(0,\Theta_0) \tag{13}$$

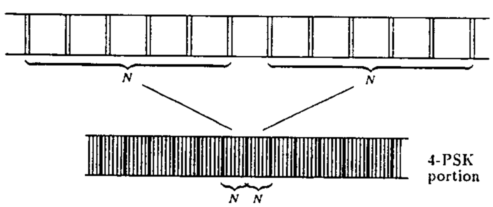
2)

$$P_f = 1 - \sum_{\Theta_0=0}^{N-1} \left[\prod_{s=1}^{s_{max}} \left(\sum_{\Theta=\Theta_0+1}^{N} P_{matr}(s,\Theta) \right) \right]^2 \cdot P_{matr}(0,\Theta_0) \tag{14}$$

Special case: $n_4 = 1$

Often there may be only one 4-PSK symbol (per modulation interval) embedded in the sequence of 8-PSK. For $n_4 = 1$ the influences of the 8-PSK and 4-PSK in equations (13) and (14) are separated.

To begin with, the 4-PSK portion is studied. The following drawing shows an extraction of the 4-PSK from the sequence:



The range of shifts in the 4-PSK portion without any overlap is given by

1) If $\lfloor N/2 \rfloor = N/2 \quad \Longrightarrow -N/2+1 \leq s_4 \leq N/2$ (15)

2) If $\lfloor N/2 \rfloor < N/2 \quad \Longrightarrow -\lfloor N/2 \rfloor \leq s_4 \leq \lfloor N/2 \rfloor$ (16)

Symbolizing the 8-PSK term by $\Pi_8(\Theta_0)$, the probability to be out of sync can be written as

1)

$$P_f = 1 - \sum_{\Theta_0=0}^{N-1} \left[\prod_{s_4=1}^{\frac{N}{2}-1} \left(\sum_{\Theta=\Theta_0+1}^{N} P_{matr}(s_4,\Theta) \right) \right]^2 \cdot \left(\sum_{\Theta=\Theta_0+1}^{N} P_{matr}(N/2,\Theta) \right) \cdot \Pi_8(\Theta_0) \cdot P_{matr}(0,\Theta_0) \tag{17}$$

2)

$$P_f = 1 - \sum_{\Theta_0=0}^{N-1} \left[\prod_{s_4=1}^{\lfloor N/2 \rfloor} \left(\sum_{\Theta=\Theta_0+1}^{N} P_{matr}(s_4,\Theta) \right) \right]^2 \cdot \Pi_8(\Theta_0) \cdot P_{matr}(0,\Theta_0) \tag{18}$$

For the 8-PSK term, one notices that $P_{matr}(s,\Theta)\,|_{8\text{-PSK}}$ is not a function of s.

$$P_{matr}(s,\Theta)\,|_{8\text{-PSK}} = \binom{N}{\Theta} 1/2^N . \tag{19}$$

With this, $\Pi_8(\Theta_0)$ follows to be

$$\Pi_8(\Theta_8) = \Big[\left(\tfrac{1}{2}\right)^N \underbrace{\sum_{\Theta=\Theta_0+1}^{N} \binom{N}{\Theta}} \Big]^{n_\Pi}$$

$$= \sum_{\Theta=0}^{N} \tbinom{N}{\Theta} - \sum_{\Theta=0}^{\Theta_0} \tbinom{N}{\Theta}$$

$$= 2^N - \sum_{\Theta=0}^{\Theta_0} \tbinom{N}{\Theta}$$

$$\Longrightarrow \Pi_8(\Theta_0) = \left[1 - \left(\frac{1}{2}\right)^N \cdot \sum_{\Theta=0}^{\Theta_0} \binom{N}{\Theta} \right]^{n_\Pi} . \tag{20}$$

n_Π remains to be determined: (see (11), (12), (15), (16)).

1) $2|N \Rightarrow 2|N(1+n_8) \Longrightarrow n_\Pi = 2\,\frac{N(1+n_8)}{2} - 1 - \left(2\frac{N}{2} - 1\right) = N \cdot n_8$

2) $2 \not| \, N$

If $2 \not| \,(1+n_8) \Longrightarrow n_\Pi = 2\lfloor N(1+n_8)/2 \rfloor - 2\lfloor N/2 \rfloor$

If $2 \mid (1+n_8) \Longrightarrow n_\Pi = 2\,\frac{N(1+n_8)}{2} - 1 - 2\lfloor N/2 \rfloor$

After the special case of $n_4 = 1$ has been treated, limits for $\sigma \to 0$ and $\sigma \to \infty$ conclude this considerations.

Limits for $\sigma \to 0$

In the case of $\sigma \to 0$ sync errors can only occur in 8-PSK portions. There, after exponentiation, +1 and −1 are equiprobable. Hence, a correlation result of N is obtained with a probability of $(1/2)^N$. Likewise, the probability of not having correlation N is given by $1-(1/2)^N$.

If all possible shifts are included, we obtain $\left(1-(1/2)^N\right)^{n_\Pi}$.

The limit for $\sigma \to 0$ results:

$$P_f(\sigma \to 0) = 1 - \left(1 - (1/2)^N\right)^{n_\Pi} . \tag{21}$$

One should note that this limit is also valid for the case of analogue correlations. Thus, for $\sigma \to 0$ P_f does not only represent an approximation, but the exact solution.

Limits for $\sigma \to \infty$

Regarding that for $\sigma \to \infty$, positive and negative correlated bits occur with a probability of 1/2, the limit of [2]

$$P_f(\sigma \to \infty) = 1 - \lim_{\sigma \to \infty} \sum_{R_c(s=0)} \prod_{s \neq 0} P(R_c(s) < R_c(s=0)) \cdot P(R_c(s=0)) \tag{22}$$

is given by

$$P_f(\sigma \to \infty) = 1 - \sum_{\Theta=0}^{N-1} \left[\sum_{i=\Theta+1}^{N} \binom{N}{i} (1/2)^N \right]^{n_s} \cdot \binom{N}{\Theta} (1/2)^N , \tag{23}$$

with n_s the total number of different shifts s.

It should be mentioned that the products in (17) and (18) may as well be approximated by summations. Considering (18) first, we obtain:

2)

$$P_f = 1 - \sum_{\Theta=0}^{N-1} \left[\prod_{s_4=1}^{\lfloor N/2 \rfloor} \left(1 - \sum_{\Theta=0}^{\Theta_0} P_{matr}(s_4,\Theta) \right) \right]^2 \cdot \Pi_8(\Theta_0) \cdot P_{matr}(0,\Theta_0)$$

$$\approx 1 - \sum_{\Theta_0=0}^{N-1} \left[1 - 2 \sum_{s_4=1}^{\lfloor N/2 \rfloor} \sum_{\Theta=0}^{\Theta_0} P_{matr}(s_4,\Theta) \right] \cdot \Pi_8(\Theta_0) \cdot P_{matr}(0,\Theta_0) \tag{24}$$

[2] R_c: cross correlation function

Approximating (17) in the same way yields

1)

$$P_f \approx 1-\sum_{\Theta_0}^{N-1}\left[1-2\sum_{s_4=1}^{\frac{N}{2}-1}\sum_{\Theta=0}^{\Theta_0}P_{matr}(s_4,\Theta)\right]\cdot$$
$$\cdot\left(\sum_{\Theta=\Theta_0+1}^{N}P_{matr}(N/2,\Theta)\cdot\Pi_8(\Theta_0)\cdot P_{matr}(0,\Theta_0)\right) \quad (25)$$

In Figure 3 probabilities P_f for some sequences have been calculated and compared with the approximation. No differences are to be seen. The corresponding limits for $\sigma \rightarrow 0,\infty$ are given subsequently in tabular form.

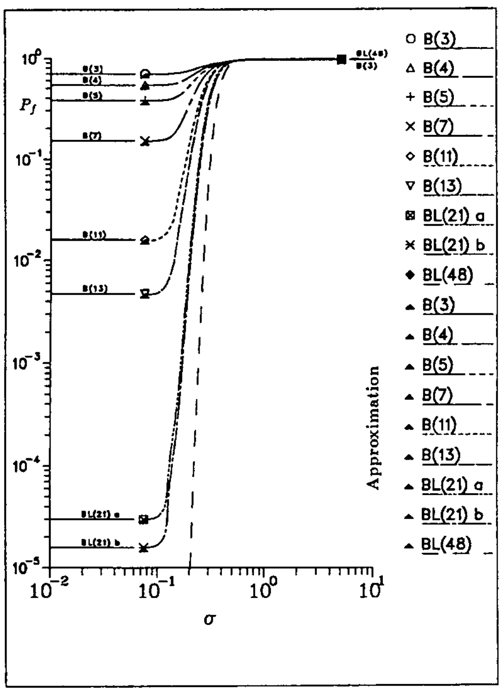


Figure 3: Figure 5: The probabilities to be out of sync P_f for time-variant 8/4-PSK in the succession '-8-8-8-4-8-8-8-4-' under additive white Gaussian noise. The superimposed sequences are Barker ones or those by Bauderon and Laubie.

Succession -8-8-8-4-8-8-8-4-

	$\sigma \rightarrow 0$	$\sigma \rightarrow \infty$
B(3)	6.9934E-01	9.7104E-01
B(4)	5.3905E-01	9.7536E-01
B(5)	3.7888E-01	9.7988E-01
B(7)	1.5186E-01	9.8394E-01
B(11)	1.5988E-02	9.8847E-01
B(13)	4.7497E-03	9.8988E-01
BL(21) a	3.0040E-05	9.9303E-01
BL(21) b	1.5736E-05	9.9328E-01
BL(48)	5.1159E-13	9.9647E-01

The reader may be surprised at the considerably high sync-error probabilities. It should be noted that real systems would probably use more than one frame to acquire sync and afterwards, when being in sync, the right sync position is assumed within a much smaller range of the shift than $[-s_{max}+1, s_{max}]$ or $[-s_{max}, s_{max}]$, respectively. Thus, the values for P_f given here will always represent the worst case. But it is easy to adapt the formulas of this contribution to the desired shift ranges.

Apart from that, the chosen discrete approximation appears to be quite useful, because also the limits for $\sigma \rightarrow 0,\infty$ equals those for an exact (analogue) computation, itself being not feasible. After some aid has been supplied to choose adequate sync-sequences, an application to time-variant, this time, block-coded modulation is presented.

Time-variant block-coded techniques have not yet been published (as far as the author knows), but following Zinoviev's description of Generalized Concatenated Codes ([10], [11], [12]), embedded 4-PSK in 8-PSK can easily be defined, even if a sync-sequence should be superimposed.

4 Time-variant block-coded 8/4-PSK based on Zinoviev's Generalized Concatenation

As the title implies, Zinoviev's construction is a generalization of the concatenation of codes. Especially, inner codes may also be defined over other metrics, such as the Euclidean metric. Thus, the alphabet of a modulation system may be regarded as an inner code. In the case of 8-PSK as inner code, for simplicity, the description in [6] is quoted: "According to the definition of GCC, the points of the 8-PSK are to be seen as an *inner code.* Three binary *outer codes* are needed to encode the three partitions of the 8-PSK given in Figure 4, each of the same length n. Written in matrix form:

$$\underline{A} = \begin{pmatrix} a^{(1)} \\ a^{(2)} \\ a^{(3)} \end{pmatrix} = \begin{pmatrix} a_1^{(1)}, a_2^{(1)}, \ldots, a_n^{(1)} \\ a_1^{(2)}, a_2^{(2)}, \ldots, a_n^{(2)} \\ a_1^{(3)}, a_2^{(3)}, \ldots, a_n^{(3)} \end{pmatrix} \begin{matrix} \longleftarrow \in \mathcal{A}^{(1)} \\ \longleftarrow \in \mathcal{A}^{(2)} \\ \longleftarrow \in \mathcal{A}^{(3)} \end{matrix} \quad (26)$$

The rows are codewords of the outer codes $\mathcal{A}^{(1)}$, $\mathcal{A}^{(2)}$ and $\mathcal{A}^{(3)}$. The columns select the corresponding points of the 8-PSK. The first component determines the 4-PSK subset, the second a 2-PSK subset of the 4-PSK set and at last, the third component decides which point of the 2-PSK set is taken. Figure 4 illustrates the procedure. The minimum quadratic Euclidean distance of two such schemes is known to be

$$d_{E_{min}} = \min_j\{d_H^{(j)} \cdot d_E^{(j)}\},$$

where $d_H^{(j)}$ is the minimum Hamming distance of the j-th outer code and $d_E^{(j)}$ is the minimum quadratic Euclidean distance between the corresponding $2^{(3-j)}$-PSK subsets."

From this description, it is simply to be seen that an embedded 4-PSK is achieved by setting components of the first row to a fixed value ('0' or '1'). This selects the 4-PSK subsets.

It follows that the sync-sequence can easily be 'superimposed' by choosing the fixed components of the first row according to the sequence itself.

The only thing that has to be mentioned is that the Hamming distance of the first outer code has to be greater by an amount of the number of fixed components, because these are to be regarded as erasures that reduce the distance of the code. For this reason, distances have to follow the relation

$$(d_H^{(1)} - n_1)d_E^{(1)} \geq \min_{j=2,3}\{d_H^{(j)}d_E^{(j)}\}.$$

where n_4 is that number of 4-PSK samples. Remarkably, interesting GCC schemes based on Reed-Muller codes given in [6] have the property that

$$d_H^{(1)} d_E^{(1)} > d_H^{(2)} d_E^{(2)}\,,\ d_H^{(3)} d_E^{(3)}\,,$$

enabling to fix one or two components of $a^{(1)}$ with nearly no loss in coding gain.

It may be expected that reducing distance in favour of improved carrier tracking ability reduces the coding gain (under perfect synchronization). A simple example given in the sequel will show that the loss is indeed very small. There the following codes are used:

(j)	(n,k,d_H)	d_E	$d_H \cdot d_E$	$\frac{d_{E_{min}}}{d_{E_{unc}}}$ / dB	R	n_4	Code
1	(14,1,14)	0.586	8.204			0	Rep.
2	(14,13,2)	2	4	3	2 / 3		Par.
3	(14,14,1)	4	4				Unc.
1	(7, 1, 7)	0.586	4.109			0	Rep.
2	(7, 6, 2)	2	4	3	2 / 3		Par.
3	(7, 7, 1)	4	4				Unc.
1	(14,1,14)	0.586	8.204			7	Rep.
2	(14,13,2)	2	4	3	2 / 3		Par.
3	(14,14,1)	4	4				Unc.

The third coding scheme, where half of its length is restricted to a fixed 4-PSK subset, reducing the distance of the first outer code to the half, is compared to the first two unrestricted schemes. Figure 5 shows the bit-error probability depending on the signal-to-noise ratio per bit for maximum-likelihood decoding. The three curves differ only slightly. As reference, theoretical and simulated curves for the uncoded 4-PSK are added.

It should be pointed out that, because of the low asymtotic coding gain of 3 dB, of course, these examples are not too interesting for applications, although soft-decision decoding is very easy. The reader is refered to [16], [6], and [7] for relevant codes with higher coding gains.

5 Conclusions

A method for frame synchronization of time-variant coded 8/4-PSK modulation has been presented. Here, the two 4-PSK subsets of the 8-PSK are alternated according to binary sync-sequences. This frame synchronization can also deliver phase information for the carrier loop and thus, combines frame and carrier synchronisation. No additional sync-headers are needed.

Probabilities for being out of sync have been derived to give some support to choose adequate sync sequences.

Furthermore, time-variant block-coded 8/4-PSK schemes have been defined. It has been shown that there is no significant loss in coding gain if some positions of the codeword sequence are restricted to 4-PSK.

Remarks:

- The methods presented here may be generalized for QAM. (For phase-invariant block-coded QAM see [17].)
- A symbol synchronization may also be included by oversampling the cross-correlation function.
- For the corresponding patent application see [18].

References

[1] **Ungerboeck, G.**: "Channel Coding with Multilevel/Phase Signals", *IEEE Trans. on Inf. Th.*, Vol. IT-28, No. 1, pp. 55-67, Jan. 1982.

[2] **Hagenauer, J., Pupeter, F., Papke, W.**: "64-kbit/s Coded 8-PSK (C8PSK) Modem Test", *Intelsat Contract INTEL-442*, March 1985.

[3] **Pietrobon, S.S., Deng, R.H., Lafanechére, A., Ungerboeck, G., Costello, D.J.**: "Trellis-Coded Multidimensional Phase Modulation", *IEEE Trans. on Inf. Th.*, Vol. 36, No. 1, pp. 63-89, Jan. 1990.

[4] **Wei, L.-F.**: "Rotationally Invariant Trellis-Coded Modulation with Multidimensional M-PSK", *IEEE JSAC*, Vol. SAC-7, Aug. 1989.

[5] **Massey, J.L.**: "Ring Codes for Phase Modulation", *Kolloquium für 'Elektronik und Nachrichtentechnik'*, ETH Zürich, Feb. 19th 1990.

[6] **Henkel, W.**: "Phase-invariant coded phase shift keying using Reed-Muller codes", submitted to *IEEE Trans. on Inf. Th.*.

[7] **Kasami, T., Takata, T., Fujiwara, T., Lin, S.**: "On Linear Structure and Phase Rotation Invariant Properties of Block *M*-PSK Modulation Codes", *IEEE Trans. on Inf. Th.*, Vol. 37, No. 1, pp. 164-167, Jan. 1991.

[8] **Hagenauer, J., Sundberg, C.-E.W.**: "Hybrid Trellis-Coded 8/4-PSK Modulation Systems", *IEEE Trans. on Comm.*, Vol. 38, No. 5, pp. 602-614, May 1990.

[9] **Bertelsmeier M., Komp, G.**: "Trellis-Coded 8PSK with Embedded QPSK", *IEEE JSAC*, Vol. SAC-7, pp. 1296-1306, Dec. 1989.

[10] **Zinoviev, V. A.**: "Generalized Concatenated Codes for Channels with Error Bursts and Independent Errors", translated from *Problemy Peredachi Informatsii*, Vol. 17, No. 4, pp. 53-56, Oct./Dec. 1981, Plenum Publishing Corporation 1982.

[11] **Zinoviev, V. A., Zyablov, V. V., Portnoy, S. L.**: "Concatenated Methods for Construction and Decoding of Codes in Euclidean Space", preprint, USSR Academy of Science, Institute for Problems of Information Transmission, 1987.

[12] **MacWilliams, F.J., Sloane, N.J.A.**: *The Theory of Error-Correcting Codes*, 4th ed., North-Holland, Amsterdam, New York, Oxford, Tokyo, pp. 590-592, 1977.

[13] **Henkel, W., Keller, P.**: "The probability of being out of frame sync for binary and complex-valued sequences", to be published in *IEE Proc. Part I*.

[14] **Gradshteyn, I.S., Ryzhik, I.M.**: *Table of Integrals, Series, and Products*, Academic Press, Orlando, 1980.

[15] **Proakis, J.G.**: *Digital Communications*, 2nd printing, McGraw-Hill, Singapore, Hamburg, London, 1985.

[16] **Sayegh, S. I.**: "A Class of Optimum Block Codes in Signal Space", *IEEE Trans. on Comm.*, Vol. COM-34, No. 10, pp. 1043-1045, Oct. 1986. (Additional table available from the author)

[17] **Henkel, W.**: "Conditions for 90°-phase invariant block-coded QAM", submitted to *IEEE Trans. on Comm.*.

[18] **Henkel, W.**: "Verfahren zur Rahmensynchronisation bei zeitvarianter codierter Phasenumtastung", *German Pat. Appl.*, P 40 18 911.2.

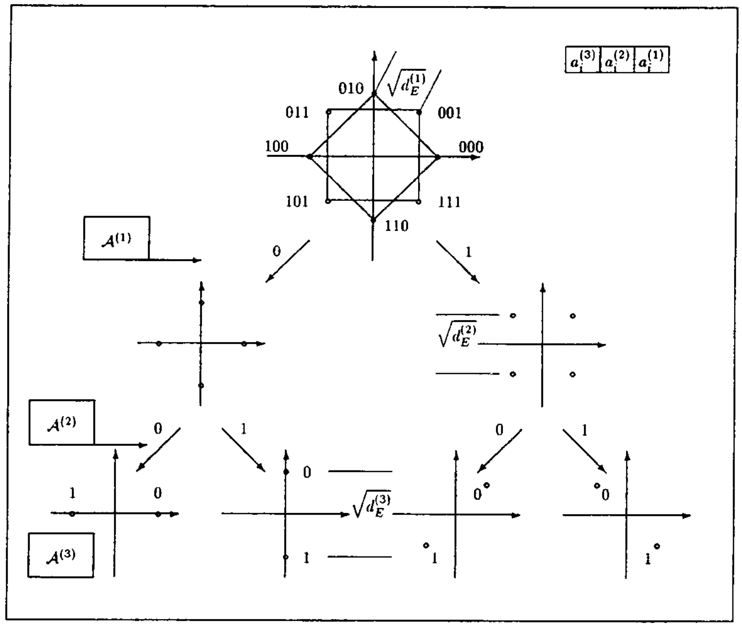


Figure 4: Set partitions of the 8-PSK

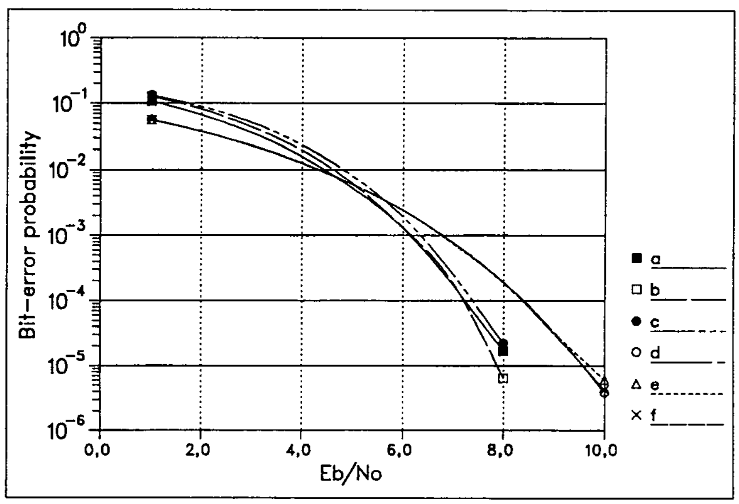


Figure 5: Bit-error probability depending on the signal-to-noise ratio per bit for the code schemes given in the table above. Curve legend: *a)* $n = 14$, $n_4 = 0$, *b)* $n = 7$, $n_4 = 0$, *c)* $n = 14$, $n_4 = 7$, *d)* uncoded 4-PSK (theoretical), *e and f)* uncoded 4-PSK (simulated)