Conclusions: The Letter has presented a newly derived spectral modulation formula for a compound PFWM hybrid pulse time modulation technique. Tests on an experimental optical fibre PFWM link have shown that excellent agreement to within ±1dB is obtained with theoretical predictions from the new modulation formula. The characteristics of PFWM render it suitable for bandwidth efficient and low-cost analogue fibre system.

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Z. Ghassemlooy and A. A. Issa (*Communication Research Group, School of Engineering Information Technology, Sheffield Hallam University, Sheffield, United Kingdom*)

B. Wilson (*Digital Communication Group, Department of Electrical Engineering and Electronics, University of Manchester Institute of Science and Technology, Manchester, United Kingdom*)

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Statistical description and modelling of impulsive noise on the German telephone network

W. Henkel and T. Kessler

Indexing terms: Subscriber loops, Telephone networks, Random noise statistics, Statistical analysis

A statistical description of impulsive noise on subscriber lines based on measurements on the network of Deutsche Bundespost Telekom is presented. Algebraic expressions for the densities of voltage, duration, and interarrival time are given. To obtain a computer model for simulations, the principles of noise generators that have the desired statistical properties are outlined.

Introduction: Impulsive noise has been the subject of many investigations. However, these surveys never led to a sufficient statistical model that could be used to define pseudo-noise generators for computer simulation. The introduction of high-speed data transmission over copper subscriber lines (HDSL: high bit rate digital subscriber line and especially, ADSL: asymmetrical digital subscriber line) has demanded new impulsive noise studies, because this type of disturbance is a major impairment for such systems.

Two recent surveys should be briefly mentioned. Work was carried out by Cook [1] at British Telecom which led to the definition of a 'symbolic pulse' which was derived from 'peak voltages' at the output of a filter bank and chosen to be balanced (DC free). Unfortunately, besides the spectral properties, it does not have a great deal in common with real measured impulses, especially because the defined pulse has a pole and infinite energy. Another measurement survey by Valenti and Kerpez [2] at Bellcore lacks a sufficiently large number of samples and had some shortcomings in the analysis of the results. However, all the mentioned studies, including that described in this Letter, are based on real measured data, not on physical considerations (see Fano [3]).

Our own survey consists of investigations at seven locations inside Germany. In each measurement, records of 50000 impulses (sampling rate 10.24MHz), histograms of the values at the ADC output with more than 10^9 samples (12bit resolution), and histograms of the interarrival times (~8×10^6 samples) were made. The statistics have been derived by curve-fitting techniques supported by self-developed interactive graphics programs.

Resulting statistics: We found that the probability density function (PDF) of samples of the impulsive noise can be approximated by

$$f_i(x) = \frac{c^5}{10\Gamma(5)} e^{-c|x|^{1/5}} = \frac{c^5}{240} e^{-c|x|^{1/5}} \quad c > 0 \qquad (1)$$

The total PDF, including the nearly Gaussian background noise, is given by

$$f_{tot}(x) = N f_n(x) + (1 - N) f_i(x) * f_n(x) \qquad (2)$$

where

$$f_n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

is the density of the Gaussian noise and $N \in [0, 1]$ is the ratio of both components. Note that the addition of Gaussian background noise results in the convolution of the corresponding densities.

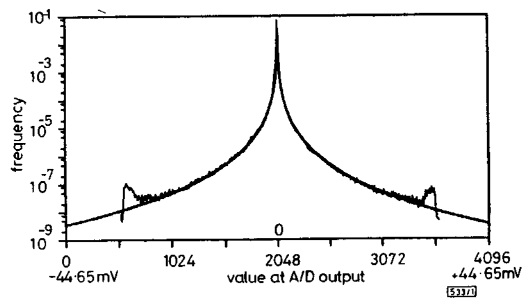

Fig. 1 *Approximation of impulsive noise density*

An approximation of a pure impulsive frequency distribution (nearly no Gaussian background noise) is shown in Fig. 1. The maxima at the left and right are due to the way the histogram was derived in this case (from recorded impulses). Actually, they are not part of the impulsive noise density.

To approximate the frequency distribution of the interarrival times, we did not consider short measured interarrival times up to ~100 μs, because they can be seen as gaps within impulses. The experimental distributions do not follow the usual Poisson density, $f(x) = \lambda e^{-\lambda x}$, because the mean impulse rate λ (~calling rate) varies over the day.

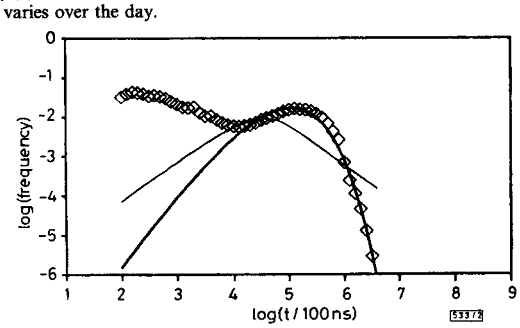

Fig. 2 *Approximation of interarrival time frequency distribution*

◊ measurement during daytime
——— average over Poisson densities with uniformly distributed λ according to Fano [2]
——— generalised Poisson density: eqn. 3, a_2 = 2.8, a_3 = 5.04, a_4 = 1.91, $x = t/100$ns

A model for the interarrival times which applies a weighted sum of Poisson densities with different λ_i (or a Markoff model) is not very suitable, because it would be too complex, due to the large number of required processes (or Markoff states). When reducing this model by using a uniform distribution of values of λ_i (see Fano [3]), we did not obtain acceptable results for all measurements (Fig. 2). Therefore, we confined our analysis to distributions of interarrival times valid only for a high calling rate during daytime (a high value of λ), i.e. to a model for the worst case. This led to a generalised Poisson density

$$f(x) = \frac{10^{a_1}}{\ln(10)} x^{a_4-1} 10^{-\frac{a_4}{\ln(a_2)} a_2^{(\log_{10}(x)-a_3)}} \quad (3)$$

which was derived by some modifications in the double logarithmic representation of the Poisson law. a_3 specifies the maximum of this log/log PDF.

The frequency distribution of the impulse lengths follows a density of log-normal type. An acceptable approximation of the density was found to be

$$\begin{aligned} f(x) =& A \frac{1}{\sqrt{2\pi}\sigma_1 x} e^{-\frac{1}{2\sigma_1^2}(\ln(x)-\mu_1)^2} \\ &+ (1-A)\frac{1}{\sqrt{2\pi}\sigma_2 x} e^{-\frac{1}{2\sigma_2^2}(\ln(x)-\mu_2)^2} \quad (4) \\ &\text{for } x > 0 \qquad (f(x) = 0 \text{ for } x \le 0) \end{aligned}$$

a sum of two log-normal densities, with fixed parameters σ_i and μ_i. Mostly, only the factor A differed significantly from measurement to measurement. In the case of a sampling rate of 10.24MHz and the corresponding sampling period, we found that $\sigma_1 = 0.75$, $\mu_1 = 4.3$, $\sigma_2 = 1.0$, $\mu_2 = 7.4$ represents most measurements.

The most demanding problem is the realisation of the spectral properties. Assuming an FIR filter of, for example, 200 taps to represent the impulse response corresponding to the average power spectral density (assuming a stationary process during the duration of the impulse), it is not feasible to construct any PDF for the input of the filter that would yield the desired output PDF by just considering the PDF transformation by the filter. Note that the characteristic function (the Fourier transform) of the PDF at the filter output would be the product of 200 stretched or shrunk versions of the input PDF:

$$F_{out}(\omega) = \prod_i F_{in}(k_i \omega) \quad (5)$$

A possible solution is to introduce a Dirac delta function into the PDF at the input of the filter. We chose

$$f_{in}(x) = \epsilon f_i(x) + (1-\epsilon)\delta(x) \quad (6)$$

with, for example, $\varepsilon \in [0.01, 0.05]$.

The Dirac approximately regenerates the desired impulse density at the output of the filter. The coefficient c of the density, of course, will change under the influence of the filter. This means that the relationship between the coefficient for the input density and that of its approximated counterpart at the output has to be computed beforehand.

The generator for the impulse samples is based on the so-called rejection method ([4], pp. 203–206 or [5], pp. 120–121), whereas the generator for the impulse lengths makes use of the transform method ([4], p. 200). The statistics of the interarrival times can be realised by applying the strip method ([6], pp. 359–368, [5], pp. 118–122).

Conclusions: Some major statistics of impulsive noise on subscriber lines derived from real measured data have been presented. The voltage density was chosen to be a double exponential with a power of 1/5 in the exponent. The interarrival time distribution was approximated by a generalisation of a Poisson density and the length distribution was proposed to be a combination of two log-normal densities. The principles of possible generators for the computer simulation of impulsive noise were outlined briefly.

(*Comprehensive report*: A more comprehensive report and an invited conference talk at Globecom '94 are currently being prepared.)

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W. Henkel and T. Kessler (*Deutsche Bundespost Telekom, Research Center, PO Box 10 00 03, D-64276 Darmstadt, Germany*)

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Error estimates and adaptive procedures for the two-dimensional finite element method

F.J.C. Meyer and D.B. Davidson

Indexing term: Finite element analysis

An *a posteriori* error estimate method, the element residual method (ERM), was investigated, and used to obtain local as well as global error estimates for finite element method (FEM) solutions of static electric field problems. The local error estimates were used to adapt the FEM meshes, leading to improved convergence rates for FEM solutions.

Introduction: Owing to the enormous speed and memory capabilities of modern-day computers, the finite element method (FEM) has been used successfully to solve electromagnetic problems which were intractable in the past. However, the solution time for the FEM grows exponentially as the size of the problem under consideration increases. The adaptive FEM considered in this Letter leads to improved convergence rates for FEM solutions, and is an attempt to use the FEM as efficiently as possible.

The FEM mesh adaptive scheme employed in this Letter uses local error estimates as criteria for identifying the regions in which the FEM mesh should be adapted. A reliable local error estimate method is thus necessary to ensure an efficient adaptive FEM. The ERM [1] is a local *a posteriori* error estimate method which has been used successfully in computational mechanics. The local error estimates can be summed over all finite elements to give a global error estimate of the FEM solution. Furthermore, a reliable and accurate global error estimate can be used as a convergence check for practical problems (for which no analytical solutions exist).

In this Letter, adaptive methods and error estimates will be applied to the FEM solution of two-dimensional static electric field problems.

FEM formulation: Consider a closed region $\bar{\Omega}$ with boundary Γ being the closure of Ω. The static electric field problem requires the solution of the Laplace equation [2] in Ω with appropriate boundary conditions on Γ. The FEM applied to the Laplace equation yields an approximate solution of the field in Ω [2].

Error estimates: Let u^1 be an approximate FEM field solution to the problem posed in the preceding Section. We define the local 'relative' error $E_k^{12} = u_k^2 - u_k^1$ as the difference between solution u^1 and an improved FEM solution, u^2, in element k. The improved