Trellis Shaping for Reducing the Peak-to-Average Ratio of Multitone Signals

Werner Henkel and Björn Wagner
Deutsche Telekom, Technology Center, 64276 Darmstadt
E-mail: henkelw@tzd.telekom.de

Abstract — **A bound for the possible reduction of the peak-to-average ratio (PAR) dependent on the rate as well as possible practical procedures are presented. The proposed methods are based on the so-called Trellis Shaping, originally published by Forney. Metrics in time and DFT domain are defined and investigated.**

I. Introduction

According to the central limit theorem the superposition of many carriers in multitone signaling leads to a Gaussian-like density. Amplifiers and A/D-, D/A-converters in transmitter and receiver have to be designed for quite high voltages, even if they might occur not too often. Simple hard limitation would result in additive noise and with it increased error rate. It would thus be advantageous to restrict the carrier-signal constellations to those with limited peak voltage in time domain. Herein, we present a limiting procedure that is based on Trellis Shaping. Beforehand, theoretical limits for the achievable PAR dependent on the rate are derived.

We assume a Gaussian density (i.i.d.) for the time-domain samples. Let the voltage limit be $\hat{s}$. The average power is

$$\bar{P} = \frac{2 \cdot \int_0^{\hat{s}} x^2 \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} dx}{2 \cdot \int_0^{\hat{s}} \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} dx} = \frac{2\sigma^2}{\frac{\sqrt{\pi}}{2}\mathrm{erf}(\frac{\hat{s}}{\sqrt{2}\sigma})} \int_0^{\frac{\hat{s}}{\sqrt{2}\sigma}} x^2 e^{-x^2} dx \,. \quad (1)$$

The probability of an amplitude lower than $\hat{s}$ is $\mathcal{P}_i = \mathrm{erf}(\frac{\hat{s}}{\sqrt{2}\sigma})$. The probability that this is the case for all N time-domain samples is $\prod_{i=0}^{N-1} \mathcal{P}_i = \mathrm{erf}^N(\frac{\hat{s}}{\sqrt{2}\sigma})$. The rate R follows to be

$$R = \frac{\log_2(2^{mN_{act}} \prod_{i=0}^{N-1} \mathcal{P}_i)}{mN_{act}} = 1 + \frac{N \cdot \log_2(\mathrm{erf}(\frac{\hat{s}}{\sqrt{2}\sigma}))}{N_{act} \cdot m} \,, \quad (2)$$

N/N_{act} being the ratio of the number of DFT components relative to the number of independently usable carriers ($N/N_{act} \approx 2$ for baseband transmission). Note that the rate is dependent on m, the number of bits per carrier, not much on N. A plot of the principal achievable PAR $= \hat{s}^2/\bar{P}$ at a certain rate R is shown in Fig. 1. Interestingly, less than 2 % of redundancy should be sufficient to reduce the PAR to the value of single-carrier QAM (or CAP, carrierless AM/PM) covering the same frequency band. However, this does not tell, how the reduction can be achieved.

II. Trellis Shaping for Peak Limitation

Unlike Forney's proposal for Trellis Shaping, we use a multi-dimensional shaper that sequentially influences the last partition in a binary partition tree. In the set partitioning of a 2^m-QAM or 2^m-PSK alphabet, this is the bit with the greatest possible change in the signal-point location. For the desired application, the DFT vector is subdivided into blocks

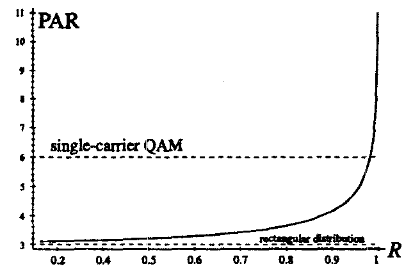

Figure 1: Achievable peak-to-average ratio (PAR) at a certain code rate R

of length n. The rate of the convolutional code is chosen to be k/n, which means an overall redundancy of $1 - R = k/(n \cdot m)$.

As optimization criteria, on one hand we discuss a metric in time domain, which is directly the peak power. This, however, does not fulfil the typical requirement for a metric inside the Viterbi algorithm, namely to be additive. The time-domain peak power has to be determined for every path segment in the trellis and the time-domain vector needs to be updated for every additional block according to

$$f_{k_\nu} = f_{k_{\nu-1}} + \sum_{l=(\nu-1)n+1}^{\nu n} F_l e^{j\frac{2\pi}{N}lk} \left(+ F_l^\star e^{j\frac{2\pi}{N}(N-l)k} \right) \,. \quad (3)$$

The complexity is at least 8 DFTs instead of 1 FFT (only of interest for broadcast applications). For a result, see Fig. 2. On the other hand, we did some investigations on a DFT-domain transition metric. Note that, unlike Parseval's formula, there is no simple expression for the peak power that would relate time and DFT domain. However, DFT-domain block-transition metrics can be tabulated and used inside the Viterbi algorithm as an additive metric. The metrics are determined as the time-domain peak power resulting from a certain block transition, setting the remaining components of the DFT frame to zero. All such possible block transitions together with the corresponding (normalized) peak powers are tabulated. This can only be applied for a small number of carriers and small QAM alphabets. The complexity for block-transition shaping is negligible.

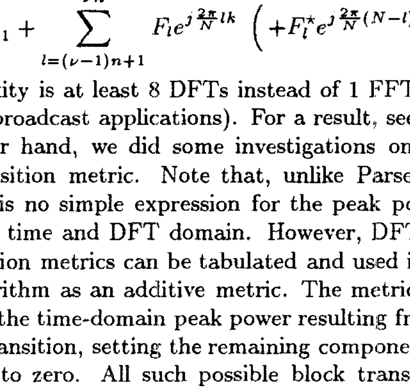

Figure 2: Histogram for shaping with time-domain metric (16-QAM, $N = 512$, shaping code rates k/n)

0-7803-3956-8/97/$10.00