A SEMI-ALGEBRAIC CONSTRUCTION TO ACHIEVE ROTATIONALLY INVARIANT CODED QAM ON THE BASIS OF MULTILEVEL CONVOLUTIONAL CODES

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1 Introduction

In order to account for carrier phase instabilities especially on satellite or mobile links, several proposals have been made to define rotationally invariant coded modulation. They were based on multidimensional or nonlinear convolutional codes, on separate encoding of the I- and Q-coordinates, or on multilevel block codes, especially with Reed-Muller codes as component codes.

This contribution describes a semi-algebraic approach with multilevel convolutional codes that leads to schemes with considerably low complexity. The construction guarantees 90°-invariance of the code, not yet of the information symbols itself. Hereto, a special differential en/decoder structure has been developed.

2 Conditions for the binary convolutional component codes

Assuming a binary set partitioning of the 2^m-QAM, with a labelling that is chosen to be 90°-invariant from the third partition label on, one obtains the following conditions:

I The all-ones sequence must be a valid code sequence of code (1). $(\ldots,1,1,\ldots,1,\ldots) \in \mathcal{A}^{(1)}$
II All valid code sequences of code (1) must be valid code sequences of code (2), too. $\mathcal{A}^{(1)} \subset \mathcal{A}^{(2)}$
III No conditions for $\mathcal{A}^{(j)}, \quad j = 3, \ldots$

3 Differential en- and decoding

The modulo-4 differential decoder is located *after* the multistage convolutional decoder. Otherwise the noise power would be doubled at the input of the differential decoder, significantly reducing the achievable coding gain.

The modulo-4 differential encoder is located *between* the encoding stages one and two (see Fig.). It can be shown that this demands for a systematic second-level code.

4 The semi-algebraic construction

As outlined in section 2, the all-ones sequence has to be a valid code sequence of code (1). For $k^{(1)} = 1$ a code with all generators having an odd weight obviously fulfills this condition.[1] For $k^{(1)} > 1$, the all-ones code sequence can be obtained, if there is the possibility of creating odd weighted generators by combining some rows (by means of the information sequence) of the Forney matrix of code (1). This, e.g., is fulfilled, if one row consists only of odd weighted generator polynomials or if the whole code is only composed of odd weighted generators.

To ensure rotational invariance for code (2), as a necessary and sufficient condition, one has to ensure that every valid code sequence $A^{(1)}$ of code (1) is also belonging to the set of code sequences $A^{(2)}$ of code (2).

$$\forall_{I^{(1)}} \exists_{I^{(2)}} : A^{(2)} = I^{(2)} \cdot G^{(2)} = I^{(1)} \cdot G^{(1)} = A^{(1)} .$$

($I^{(j)}$: Info series, $G^{(j)}$: Forney generator matrix)
As this equation has to be fulfilled for arbitrary $I^{(1)}$, $I^{(2)}$ appears as a function of $I^{(1)}$. A possible approach for the construction of code (2) is to define the components of $I^{(2)} = (I_1^{(2)}, I_2^{(2)}, \ldots, I_{k^{(2)}}^{(2)})$ as shifted versions of $I^{(1)}$ (assuming $k^{(1)} = 1$):

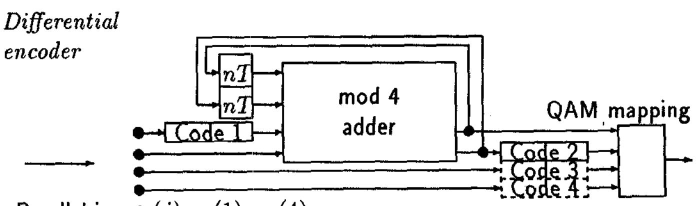
$$I_h^{(2)} = I^{(1)} \cdot D^{j_h} \quad (k^{(1)} = 1, h = 1, \ldots, k^{(2)}, j_h \in \{0, \ldots, L^{(1)} - 1\}) .$$

$L^{(j)}$ is the constraint length of code (j) (not multiplied with $k^{(j)}$). D is a time delay factor (z^{-1} of the Z-transform).

There has to be at least one $I_h^{(2)} = I^{(1)}$, i.e. $j_h = 0$, in order to express the low-order term $D^0 = 1$, appearing in $G^{(1)}$, by means of $G^{(2)}$. Furthermore, one j_h has to equal $j_h = L^{(1)} - L^{(2)}$. This is necessary as the term $D^{L^{(1)}-1}$ appearing in $G^{(1)}$ has to be expressed by $G^{(2)}$ with the maximum exponent $L^{(2)} - 1$. For reasons of decoding complexity it is useful to have $L^{(2)} \leq L^{(1)}$, because $k^{(2)}$ is usually greater than $k^{(1)}$. This can be achieved by the proposed construction leading to a considerably low decoding complexity.

Some results are given subsequently. A coding scheme with an asymptotic coding gain of 6 dB, e.g., has a complexity of 4 states for the first stage and 8 states for the second (and, maybe, additionally the Wagner decoding of a parity-check code as a third stage).

	Code (1)	Code (2)
Gen. non-rec.	(4,7,7)	(10,13,15), (16,13,15)
$R^{(j)}, L^{(j)}, d_f^{(j)}$	$\frac{1}{3}$, 3, 6	$\frac{2}{3}$, 2, 3
Gain / dB	4.7	
Gen. non-rec.	(15,15,13)	(51,61,73)
$R^{(j)}, L^{(j)}, d_f^{(j)}$	$\frac{1}{3}$, 4, 9	$\frac{2}{3}$, 3, 5
Gain / dB	6.5	
Gen. non-rec.	(1,2,7,7)	(46,52,61,73)
$R^{(j)}, L^{(j)}, d_f^{(j)}$	$\frac{1}{4}$, 3, 8	$\frac{3}{4}$, 2, 4
Gain / dB	6	



[1] $k^{(j)}$: number of info bits per frame, coderate $R^{(j)} = \frac{k^{(j)}}{n}$