Maximizing the Channel Capacity of Multicarrier Transmission by Suitable Adaptation of the Time-Domain Equalizer

Werner Henkel, *Member, IEEE,* and Thomas Kessler

Abstract—An adaptation algorithm for determining the time-domain equalizer coefficients is described that maximizes the total channel capacity for all carriers of a multitone (discrete multitone) transmission. It takes into account the crosstalk noise environment and the interblock interference as a common disturbance. Furthermore, the leakage effect of the discrete Fourier transform (fast Fourier transform) is considered, too. Including this into the algorithm for the equalizer coefficients leads to a notable improvement in the signal-to-noise ratio, especially at lower frequencies for a typical asymmetrical digital subscriber line application.

Index Terms—DMT, guard interval, OFDM, TEQ, time-domain equalizer.

I. INTRODUCTION

MULTITONE transmission is usually realized by an inverse fast Fourier transform (IFFT), where the discrete Fourier transform (DFT) vector represents the carriers with signal points of (different) quadrature amplitude modulation (QAM) alphabets on it. In order to simplify the equalization, a so-called guard interval is provided in time domain, which is a cyclic prefix. This enables to regard the convolution by the channel as a cyclic convolution as long as the impulse response of the channel is limited to the length of the guard interval. A cyclic convolution in time domain is equivalent to a multiplication of the components in DFT domain with complex factors. This means that an equalizer can be realized by dividing by these factors, which is an automatic gain control (AGC) for every frequency component. Thus, equalization is quite simple, as long as the impulse response meets the length restriction. However, if this is not the case, a time-domain pre-equalizer has to be provided to shorten the impulse response. According to the asymmetrical digital subscriber line (ADSL) standard, e.g., a guard interval of only 32 samples has been specified, whereas a typical channel-impulse response can span more than 200 samples. For coefficient adaptation of the time-domain equalizer, several different methods have been proposed. These can be classified as three principal procedures, which will be described in the next section. Afterwards, the new algorithm will be outlined.

II. PRINCIPLES FOR COEFFICIENT ADAPTATION OF A DISCRETE MULTITONE (DMT) TIME-DOMAIN EQUALIZER

Procedure I: The simplest way to come to a coefficient setting is to approximate the channel response in $H(D)$ by a rational function

$$H(D) = A(D)/B(D). \tag{1}$$

An equalizer with a response $W(D) = B(D)$ reduces the total response to $A(D)$. Such an approximation, which can, e.g., be carried out by using *invfreqz* from the Signal Processing Toolbox of Matlab, shows that the degree of $A(D)$ can easily be kept within the length of the guard interval (up to 4 km of twisted pairs with a wire diameter of 0.4 mm). The advantage of the method is that routines are already available and it can thus be easily applied. However, the criterion that the resulting impulse response should be at most as long as $n_G + 1$, where n_G is the guard-interval length, is only represented by the choice of the degree of the polynomial $A(D)$. The most important criterion, which is the maximization of the geometric mean of the signal-to-noise ratios (SNRs) for the used carriers, is not considered. This criterion results from the maximization of the sum R over the channel capacities R_i of the N_{carr} carriers

$$R = \sum_{i=0}^{N_{\text{carr}}} R_i = \sum_{i=0}^{N_{\text{carr}}} \log_2 \left(1 + \frac{S_i}{N_i} \right) \tag{2}$$

being a measure for the maximum bit content. S_i denotes the average signal power at carrier location i and N_i is the corresponding average noise power. For high SNRs the "1" could be neglected, which means that an optimization of the geometric mean of the SNRs is carried out. When restricting the summation in (2) to such carriers with higher SNRs, this means that the geometric mean of the SNRs could be maximized instead of the channel capacity. One may object that, anyway, the channel capacity is not the right parameter to be optimized. For practical systems, indeed, the relation between bit-error rate (BER) and achievable data rate should be considered. However, a BER can only be computed after bits have been allocated to the carriers. Furthermore, experience with, e.g., different bit-allocation algorithms based on error rate or channel capacity indicates that performance differences are quite small.

Up until now, it seems that two criteria have to be taken into consideration as follows.

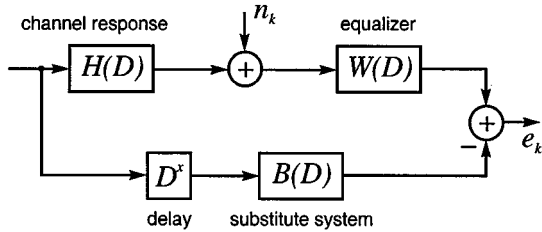1) The impulse response should be shorter than the guard interval (at most as long as $n_G + 1$).

Fig. 1. Substitute system of channel-impulse response and time-domain equalizer as a reference.

2) The geometric mean of the SNRs S_i/N_i at the used carrier locations should be maximized.

Procedure II: The second principal procedure uses a reference or substitute system that represents the combination of channel and equalizer (see Fig. 1). The order of this reference system (transversal filter) equals the desired impulse-response length. The mean-square error between the output of the substitute and the output of the equalizer has to be minimized. This procedure has been proposed in several publications all originating from [1] (e.g., [2]–[4]) and leads to an eigenvalue problem with correlation matrices. However, there exist also some simplifications based on iterative cutting operations. Note that in Fig. 1 the noise power spectral density (PSD) is taken into account through n_k. The procedure is still not optimum, since it does not optimize the sum of channel capacities or the geometric mean of the SNRs. Nevertheless, it often leads to quite acceptable results.

Procedure III: Both criteria 1 and 2, together, were first treated in a paper by Al-Dhahir and Cioffi [5]. Unfortunately, after a simplification, only a geometric mean of the channel-response components is maximized, and in order to take the noise into account in some way, a mean-square error approach like the one of Procedure II is used to introduce an additional optimization constraint. A nonlinear constrained optimization algorithm from the Optimization Toolbox of Matlab is applied. Although the original approach of maximizing the geometric mean of the SNRs has not really been pursued consistently, the beginning of the paper pointed in the right direction.

III. The New Algorithm

For an optimization routine, it would be an advantage to have only one parameter that has to be maximized or minimized. This means that both criteria need to be combined into one. Anyway, both represent noise components that should be minimized. It should be noted that interblock interference caused by an impulse response exceeding the guard interval has to be considered as a noise component.

The new algorithm optimizes only one parameter, namely R given in (2), which means (approximately) the maximization of the geometric mean of the SNRs. All disturbances are taken into account, both external noise like crosstalk and interblock interference.[1] Very important is that the leakage effect of the DFT (FFT) is taken into consideration, too. An optimum is found that does not necessarily restrict the total impulse response to the length of the guard interval.

[1]Intercarrier interference is not considered.

When performing the DFT transform in a DMT receiver, a rectangular time-domain window is used. This has no meaning for the data signal which has a cyclic prefix. For noise signals and also data-signal components that lead to interblock interference, this is not true. No cyclic prefix is available for such disturbances. Hence, the effect of the rectangular windowing, called "leakage," has to be computed inside the algorithm.

In the sequel, the steps of the iterative algorithm are described. There, N_{carr} denotes the maximum possible number of carriers. For baseband applications like DMT, one has to consider conjugacy constraints to ensure that the time-domain signal is real. This means that the upper half of the DFT frame is dependent on the lower half. The DFT block length is then $N_{\mathrm{DFT}} = 2 \cdot N_{\mathrm{carr}}$. The algorithm, however, requires an oversampling in frequency domain by a factor of two, which leads to a DFT block length of $N_{\mathrm{DFT2}} = 4 \cdot N_{\mathrm{carr}}$.

The Steps of the Adaptation Algorithm:

1) Determine the discrete noise PSDs $\mathcal{N}_i^2$, $i = 0, 1, \ldots, 4N_{\mathrm{carr}} - 1$.

2) Let $\mathcal{H}_i$ be proportional to the discrete channel response. The quotient $\mathcal{H}_i/\mathcal{N}_i$ is normalized, such that $\sum_i |\mathcal{H}_i|^2 / \sum_i |\mathcal{N}_i|^2$ represents the SNR, denoted $\mathrm{SNR}_{\mathrm{in}}$, at the input of the equalizer when using all possible carriers with the same transmit power:[2]

$$\mathcal{H}_i := \mathcal{H}_i \cdot \sqrt{\mathrm{SNR}_{\mathrm{in}} \cdot \frac{\sum_i |\mathcal{N}_i|^2}{\sum_i |\mathcal{H}_i|^2}} . \qquad (3)$$

3) Product $\mathcal{S}_i$ of the channel response $\mathcal{H}_i$ times the equalizer response W_i in the DFT domain

$$\mathcal{S}_i = \mathcal{H}_i \cdot W_i. \qquad (4)$$

4) Determine the corresponding impulse response by applying the IFFT

$$s = \mathrm{IFFT}(\mathcal{S}). \qquad (5)$$

5) Determine the mean signal power density fraction $|N_i^s|^2$ of the part of the impulse response s that exceeds the guard interval and influences the next frame.

6) Compute the product of $\mathcal{N}_i^2$ times the square of the discrete frequency response of the equalizer W_i

$$(N_i^n)^2 = \mathcal{N}_i^2 \cdot W_i^2. \qquad (6)$$

7) Add the noise power density components from steps 5 and 6

$$|N_i|^2 = |N_i^s|^2 + |N_i^n|^2. \qquad (7)$$

8) Compute the autocorrelation function n_i that corresponds to the total noise PSD $|N_i|^2$

$$n = \mathrm{IFFT}(\langle |N_i|^2 \rangle) \qquad (8)$$

$$\langle |N_i|^2 \rangle = (|N_0|^2, |N_1|^2, \ldots, |N_{4N_{\mathrm{carr}}-1}|^2).$$

[2]The assumption of constant transmit power density is typical for xDSL applications at high SNR. However, an arbitrary frequency dependency of the transmit PSD may be realized by modifying the channel response $\mathcal{H}_i$, i.e., including the PSD frequency dependency in the channel frequency response.
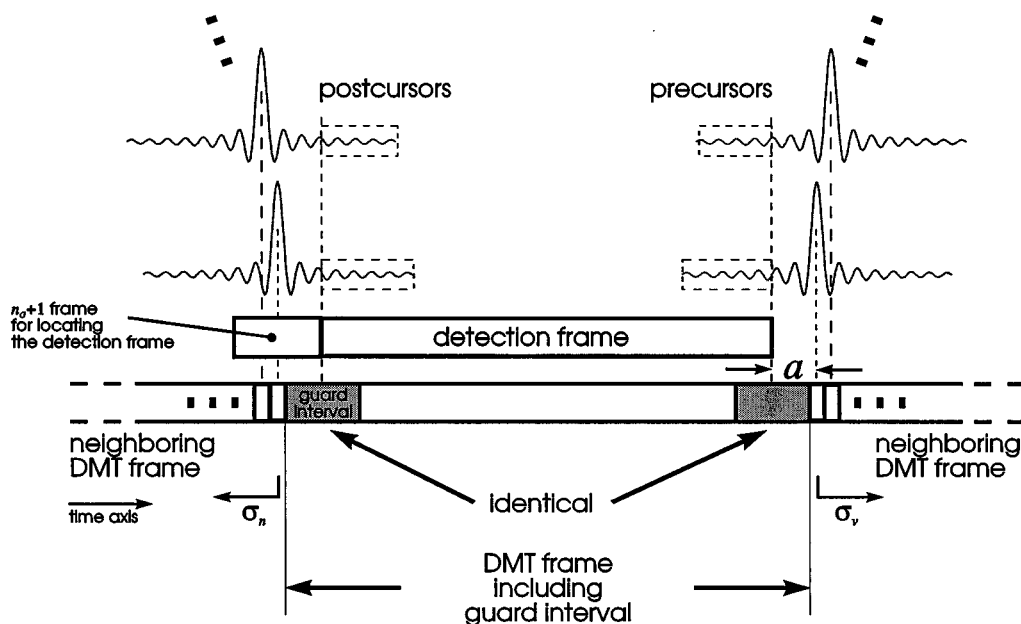
Fig. 2.   Interblock interference from neighboring frames.

9) Introduce the leakage effect of the DFT by multiplying n_i by a triangular function t_i

$$\lambda_i = n_i \cdot t_i, \qquad i = 0, \ldots, 4N_{\mathrm{carr}} - 1 \qquad (9)$$

$$t_i = \begin{cases} \dfrac{2N_{\mathrm{carr}} - i}{2N_{\mathrm{carr}}}, & i = 0, \ldots, 2N_{\mathrm{carr}} \\[2ex] \dfrac{i - 2N_{\mathrm{carr}}}{2N_{\mathrm{carr}}}, & \\ \qquad i = 2N_{\mathrm{carr}} + 1, \ldots, 4N_{\mathrm{carr}} - 1. \end{cases}$$

10) According to [6], the influence of a rectangular time-domain windowing, which is due to the DFT operation at the receiver, corresponds to the multiplication of the time-domain equivalent (autocorrelation function) of the noise PSD times a triangular function. In order to describe the triangular function approximately with the discrete time-domain vector, an oversampling in frequency domain by (at least) a factor of 2 is chosen. This means that the algorithm relies on a frequency scale with at least the half of the DMT carrier spacing.

11) Transform of λ into the DFT domain

$$\Lambda = \mathrm{FFT}(\lambda). \qquad (10)$$

12) Determine the SNR, denoted SNR_i, for every used carrier location. No frequencies are considered that just resulted from the oversampling, only the ones that are actually used and are at multiples of the original carrier spacing. We neglect the reduction of the signal power by the interblock interference.

$$\mathrm{SNR}_i = |\mathcal{S}_i|^2 / \Lambda_i, \qquad i \in \mathtt{U} \qquad (11)$$

U denotes the set of actually used carrier locations.

13) Summation of all channel capacities at all carrier locations

$$R = \sum_{i \in U} \log_2(1 + \mathrm{SNR}_i). \qquad (12)$$

R is the parameter to be optimized, which can be carried out by any multidimensional optimization algorithm, like AMOEBA (downhill simplex method) [7] or differential evolution [8], iteratively modifying the time-domain equalizer coefficients. For each iteration, Steps 3–13 have to be repeated. The change of R between two successive iterations may serve as a termination criterion. This means, the actual value of R is stored for the next iteration to allow for a comparison, afterwards. If the change of R falls below a certain threshold, the iteration is terminated.

Instead of R, the geometric mean of all the SNR_i, $i \in \mathtt{U}$, can be maximized.

The two noise components, external noise and interblock interference, are combined in Step 7 and the leakage is treated in Step 9.

Step 5 needs to be explained in some more detail. When receiving a DMT signal, the detection frame will be positioned in time such that the interblock interference is minimized. The optimum position is a tradeoff, where postcursors resulting from components of the preceding frame as well as precursors resulting from components of the following frame have to be considered as a disturbance. Four such exemplary impulse responses resulting from components of neighboring frames next to the detection frame are depicted in Fig. 2 (for illustration purposes, in continuous form). The rectangular dots symbolize further impulse responses resulting from the following components on both sides of the detection frame. For every position of the impulse responses, the portion inside the detection frame has to be extracted by a rectangular window. The number of impulse responses that have to be considered is dependent on the length of the impulse response. Applying an FFT and afterwards computing the squared amplitudes of every DFT-domain component (periodogram) yields the corresponding noise PSD. All such noise PSD contributions from the neighboring frames have to be summed up to obtain a mean noise PSD of the interblock interference. Like for

signal-independent external noise, the rectangular windowing of the DFT has to be considered. This means the leakage effect comes into play here, too. For illustration, both noise types are summed up in Step 7, before the leakage is incorporated in Step 9. Hereto, the detection frame has to be taken twice as long, like for external noise. However, the computationally more efficient solution would be to use a nonextended detection frame, whereby the leakage is automatically taken into account. The less computationally efficient description has been preferred here, because it unveils the similarity between both noise components more clearly.

Let the length of the guard interval be n_G. The position of the detection frame can be estimated by first finding the position of a frame of length $n_G + 1$ with the maximum energy content when sliding over the entire impulse response. The detection frame starts right after it.

In the sequel, the procedure is again described in a more formal way. Let us first suppose that the time axis origin of the total impulse response s_i is located at the "center of energy." Such a time axis shift has no influence on the results. Now, let the impulse response have nonzero components for $i = -n_v, \ldots, 0, \ldots, n_n$, with n_v being the number of precursor and n_n being the number of postcursor samples. The mean PSD portion $|N_i^s|^2$ resulting from the part of the impulse response that influences the neighboring frames can be computed as follows:

Precursors: For all $\sigma_v = 0, \ldots, n_v - a$ (possible positions of the impulse response, resulting from different samples; parameter a, see, Fig. 2)

$$s^{(\sigma_v)}_{4N_{\mathrm{carr}}-1-i} = s_{-a-i-\sigma_v}, \quad i = 0, \ldots, n_v - a - \sigma_v$$

$$s^{(\sigma_v)}_{4N_{\mathrm{carr}}-1-i} = 0, \qquad i = n_v - a - \sigma_v + 1, \ldots,$$
$$4N_{\mathrm{carr}} - 1.$$

Postcursors: For all $\sigma_n = 0, \ldots, n_n - n_G + a - 2$

$$s^{(\sigma_n)}_i = s_{n_G - a + 1 + i + \sigma_n}, \quad i = 0, \ldots, n_n - n_G + a$$
$$-2 - \sigma_n$$

$$s^{(\sigma_n)}_i = 0, \qquad i = n_n - n_G + a - 1 - \sigma_n, \ldots,$$
$$4N_{\mathrm{carr}} - 1.$$

As already described, we chose the frame length of the detection frame to be $4N_{\mathrm{carr}}$ samples long, because of similar treatment of both noise components. The real detection frame at a DMT receiver has, of course, only $2N_{\mathrm{carr}}$ samples.

The mean PSD of the interblock interference is now obtained by summing up all precursor and postcursor components.

$$|N_i^s|^2 = \sum_{\sigma_v} |\mathcal{S}_i^{(\sigma_v)}|^2 + \sum_{\sigma_n} |\mathcal{S}_i^{(\sigma_n)}|^2$$

with $\mathcal{S}^{(\sigma_v)} = \mathrm{FFT}(s^{(\sigma_v)})$, $\mathcal{S}^{(\sigma_n)} = \mathrm{FFT}(s^{(\sigma_n)})$.

In a rigorous sense, this sum is only correct if all components are uncorrelated, which is only true if all carriers are really used with statistically independent random data. Thus, the sum should be regarded as an approximation. Also, applying the triangular function in Step 9 is based on the assumption of
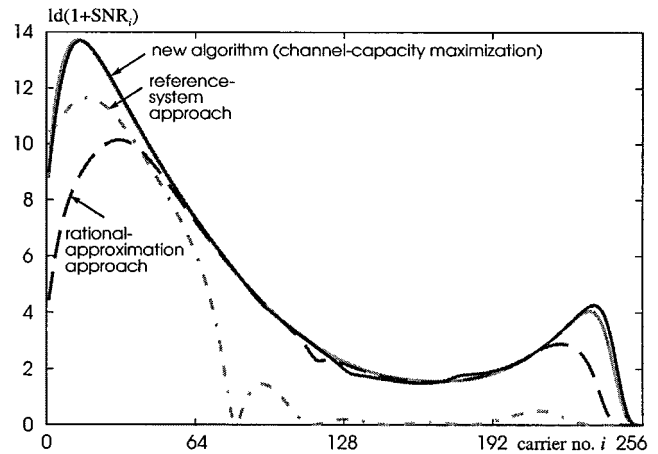

Fig. 3. SNRs (all carrier locations down to the first are shown, although, of course, the first five carriers cannot be used due to POTS transmission and the splitter-filter transition region).

stationarity, which is not fulfilled for the tails of the impulse responses that represent the interblock interference. Nevertheless, actual DMT transmission simulation results are in good agreement with the SNRs computed in the algorithm.

Since in our algorithm the optimization of the sum of channel capacities is restricted to such carrier positions that are really used for transmission, one may realize that this set of carriers $\mathbb{U}$ may change after bit allocation. Thus, one could, in principle, think of a combination of the algorithms for the equalizer coefficients and the bit allocation. However, we have found that small changes in $\mathbb{U}$ do not cause relevant changes in the setting of the equalizer coefficients.

The disadvantage of the algorithm is its quite high complexity. $4 + (n_v + n_n - n_G)$ FFTs are required per iteration. The number in brackets is due to Step 5 with FFTs over the sparse vectors $s^{(\sigma_v)}$ and $s^{(\sigma_n)}$.

IV. RESULTS

Our simulation results show the advantages of the new algorithm. An equalizer with a coefficient setting according to the new procedure when applied for a 2-Mb/s ADSL (ANSI T1.413) link with underlying POTS transmission and a quite extreme noise environment with eight high-rate NEXT sources (digital loop carriers) within the same basic bundle of the cable (0.4 mm) lead to a reach improvement from 3.3 (rational approximation approach) to 3.5 km. The number of iterations required is 100–200. Figs. 3 and 4 show the resulting SNRs (log scale!) and the impulse responses, respectively. There, the new algorithm is initialized with the solution of the rational-approximation approach or the reference-system approach. Fig. 3 illustrates the advantage in SNRs. The two dashed lines show the performances with the initial equalizer settings, whereas the solid lines are the corresponding results after applying the new algorithm, maximizing the channel capacity. The rational-approximation approach showed shortcomings especially at low frequencies, whereas the reference-system approach was somewhat better there, but was generally inferior to the new approach for all frequencies. Nevertheless, there exist manifold alternative descriptions of the reference-system approach, of which one
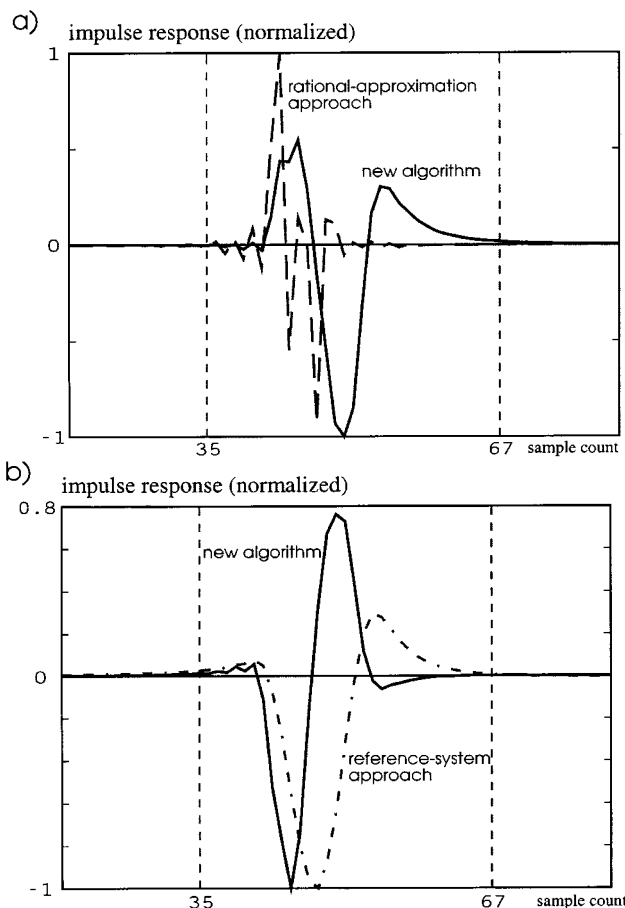
a)

b)

Fig. 4.   Impulse responses after time-domain equalizing when starting the new algorithm from the result of (a) the rational-approximation approach or (b) the reference-system approach.

or the other may yield better results. Our realization of the reference-system approach followed mostly the description in [2]. In time domain (see Fig. 4), both the reference-system approach and the newly proposed method show an effective use of the available guard interval, whereas the impulse response resulting from the rational approximation is far too short. What we also see very clearly from Fig. 4 is that depending on the initialization, the new algorithm may end up in a totally different solution, which, however, has been experienced to have comparable and always much better performance than with the initialized coefficients, which can also be seen from the two solid lines in Fig. 3. The existence of local maxima and, thus, the dependence on the initialization are typical for such nonlinear optimization problems. However, for ADSL parameters, we only saw convergence to well-performing coefficient sets. As a further information, an exemplary adaptation curve is provided in Fig. 5.

V. CONCLUSIONS

We described a new algorithm for determining the coefficients of a DMT time-domain equalizer. It optimizes the
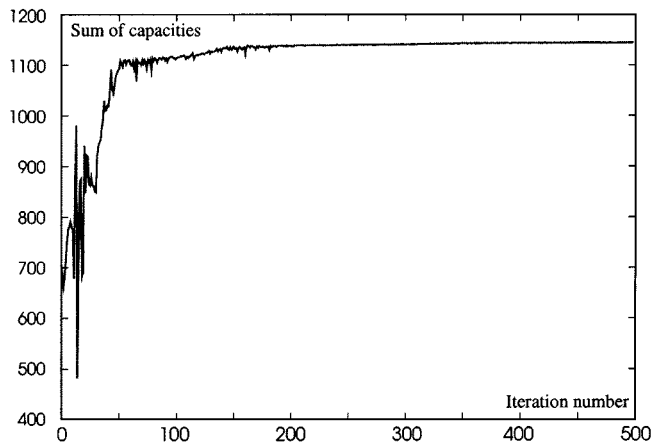

Fig. 5.   Example of an adaptation curve.

channel capacity and takes into account interblock interference and external noise. Additionally, the effect of the rectangular windowing due to the FFT at the DMT receiver is considered. Simulation results showed the superior performance of the new algorithm compared with rational-approximation and reference-system approaches. A drawback of the method is the high computational complexity which makes it rather slow relative to the DMT frame duration.

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