Multiuser MIMO-OFDMA with Different QoS Using a Prioritized Channel Adaptive Technique

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***Abstract*—In this paper, an orthogonal multiple access scheme is considered for different users with different quality of service (QoS) requirements. Therefore, the users are multiplexed using orthogonal frequency division multiple access (OFDMA) transmission. A multiple-input multiple-output (MIMO) antenna system is considered between the base-station (BS) and the existing mobile sets (MSs). In order to realize different QoS, the transmission bit-rates and powers are adapted to the various channel conditions with different margin separations. In here, we proposed two different subcarrier sorting schemes in order to exploit the multiuser diversity and to enhance the performance in case of channel uncertainties. Accordingly, our simulations examine the performance of the downlink (broadcast) channel adaptation for different channel conditions assuming a single cell scenario.**

***Index Terms*—MIMO, OFDMA, adaptive modulation, power allocation, limited feedback, QoS, multiuser diversity**

I. Introduction

Multiuser communication aims at sharing the resources amongst a number of users. Usually, these users require different levels of protection according to their applications and/or their class of services. Strictly speaking, these users can be ranked according to their QoS requirements. Accordingly, OFDMA is selected as a transmission system. On the one hand, OFDMA has a fine frequency granularity which exploits the multiuser diversity, thereby, enhance the spectral efficiency. On the other hand, it could simply realize different QoS requirements by adapting the transmission parameters to the different channel state information (CSI) of each user according to his performance constraint. This is known in the single user communication as unequal error protection (UEP) channel adaptation [2], where different performances are devoted to different clusters of the transmitted data according to the required symbol-error ratio (SER) by exploiting the different channel conditions across the frequency subcarriers.

Additionally, applying multiple antenna arrays at each transceiver extends the adaptation freedom to the spatial domain as well. This could be simply realized by considering proper pre- and post-processing matrices [12]. However, in this case, the accuracy of the CSI becomes very critical. Generally speaking, a perfect CSI at the BS is a practically unrealistic assumption. Nevertheless, a partial CSI may be sufficient to maintain a certain performance [9]. In order to solve this problem in a multiuser environment with different QoS classes, we introduce a novel adaptation scheme that keeps arbitrary performance margins between users to satisfy a certain QoS.

Historically, in single-link single-priority communication, a number of algorithms have been introduced to adapt the multicarrier systems. Hughes and Hartogs [3] have proposed a discrete margin adaptive bit-loading algorithm that successively allocates bits to subchannels that require the minimum incremental power. The bit-rate adaptive scheme considered by Chow et al. in [4] allocates bits according to their signal-to-noise ratios (SNR) and the required target bit-rate making use of Shannon's capacity formula by introducing an adaptive noise margin γ. These algorithms are known from wireline literature. Therefore, the channel in these cases is assumed to be perfectly known at the transmitter. However, in todays wireless communications, the computational complexity and the CSI feedback rate have to be significantly reduced.

In this paper, we extend the work in [1] in order to adapt the transmission for a number of prioritized users using a simple subcarrier sorting and partitioning method. Additionally, the algorithm's core steps have been simplified more to suite the multiuser wireless mobile systems. As in [9], a CSI subject to errors/delays is seen as a serious threat for preserving the decomposed MIMO eigenchannels orthogonality [8]. However, multiuser diversity, due to the channel randomness [10], increases the chance of exploiting the stronger subchannels only. Thereby, it is more susceptible to harmfull CSI variations.

A frequency division duplexing (FDD) is assumed with a separate CSI feedback channel, which leads to CSI errors. The pre- and post-processing matrices for our MIMO channels (for each user's subcarriers) are derived from a singular value decomposition (SVD) in order to produce equivalent multiple parallel single links (eigenchannels) [12]. Since OFDMA is assumed, any subcarrier (including all its eigenchannels) must be fully dedicated to a single user. Therefore, to realize different QoS classes amongst the given users, the subcarriers of each user have to be sorted according to their eigenchannels using one of the following proposed criteria:

- maximizing the channel capacity (full beamforming),
- avoiding the inter-eigen (single beamforming).

Finally, this paper is organized as follows. Section 2 discusses the multiuser MIMO-OFDM system model. Section 3 introduces our QoS adaptive modulation using a Chow-like channel adapation method. Section 4 discusses the results. Finally, we conclude our findings in the last section.

II. Multiuser MIMO-OFDM Adaptive Model

The MIMO-OFDMA combination is selected due to its capability for adapting both the frequency and the spatial domains. Moreover, two different subcarrier sorting criteria have been proposed to utilize the multiuser diversity and maximize the multiplexing gain efficiently. In the following section, we will introduce the main channel parameters and considerations.

A. Channel Model

Herein, we consider a downlink transmission scenario, where the BS is equipped with N_T transmit antennas and K MSs, each with N_R receive antennas. This forms the MIMO channel matrices $\mathbf{H}_{k,u} \in \mathbb{C}^{N_R \times N_T}$, where u is the user index and k is the subcarrier index of the OFDMA frame assuming N total subcarriers. We further assume a partial CSI with a quantized/outdated channel $\overline{\mathbf{H}}_{k,u}$ which deviates from the instantaneous channel $\mathbf{H}_{k,u}$ by a total error $\Xi_{k,u}$. This error is simply defined as $\Xi_{k,u} = \mathbf{H}_{k,u} - \overline{\mathbf{H}}_{k,u}$, where $\Xi \in \mathcal{C}\ \mathcal{N}(0, \sigma^2_{\Xi}\mathbf{I})$ [5]. Channel matrix entries are modeled as independent Rayleigh fading blocks composed of L different paths (echoes); each path has its own amplitude β_l, delay τ_l, and random uniform phase shift $\theta_l \in [0, 2\pi)$. The time-variant channel impulse response for each channel matrix element, can be defined following [6] as

$$h^{n_r,n_t}(t) = \sum_{l=0}^{L-1} \beta_l(t) p_l(t) e^{j\theta_l t} \delta(t - \tau_l) \ , \tag{1}$$

where t is the observation time, β_l is an i.i.d. zero mean random complex Gaussian variable, and p_l is an exponentially decaying factor. Therefore, one can assume that the elements of $\mathbf{H}_k$ (in frequency domain) are also i.i.d. with a zero mean circularly symmetric complex Gaussian distribution; i.e., neglecting the subcarrier correlation which is tied to L. Therefore, assuming no antenna correlation, $\mathbf{H}_k \in \mathcal{C}\ \mathcal{N}(0, \sigma^2_{\overline{\mathbf{H}}}\mathbf{I})$ [5], where these randomness ensures multiuser diversity exploitation as well.

B. System Model with Limited CSI Regime

Figure 1 depicts a multiuser-multiantenna system with different QoS levels receiving a combined OFDMA broadcast frame from a single BS through a MIMO downlink channel. The users' MSs are assumed to be randomly located around the BS, however, the impact of the large scale fading (path-loss and shadowing effect) is implicitly considered in the signal-to-noise ratio (SNR) calculated at the receiver front end. Further details like relative MSs distances and transmit power control (TPC) are assumed to be ideally treated and out of the scope of this paper.

In the OFDMA transmission, each user is entitled to transmit over a non-overlapping set of his stronger subcarriers. This ensures that the maximum SNRs are always utilized to guarantee maximum multiuser diversity exploitation. In order to avoid interference (due to the channel delay spread), an appropriate cyclic-extension/guard-interval is assumed to be added at each OFDMA frame to mitigate both inter-frame interference and inter-carrier interference (inter-user interference at the user boundaries) [11].

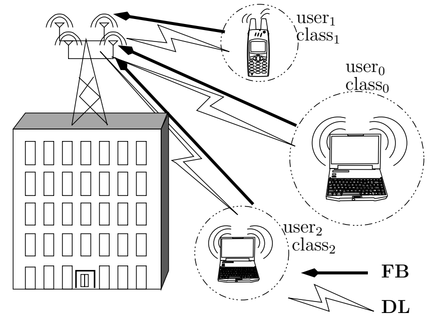

Fig. 1. Multiuser MIMO-OFDM with different QoS, where user_0 has the highest QoS (class_0). Each user receives a downlink (DL) transmission from the BS and transmit a feedback (FB) signaling on the uplink

Finally, for each subcarrier k, each user u transmits his quantized feedback CSI ($\overline{\mathbf{H}}_{k,u}$ as mentioned before) with a certain propagation and processing delay. To simplify our notations in this section only, we will discard the subscript $\{k, u\}$.

To preform channel adaptation in MIMO, one has to consider decomposing the MIMO channel into a number of parallel (non-interfering) single channels using eigenvalue decomposition (EVD) in case of Hermitian matrices or singular value decomposition (SVD) [7]. Therefore, the Hermitian $\overline{\mathbf{H}}^{\mathrm{H}}\overline{\mathbf{H}}$ can be decomposed (using EVD) as follows

$$\overline{\mathbf{H}}^{\mathrm{H}}\overline{\mathbf{H}} = \overline{\mathbf{V}}\ \overline{\mathbf{D}}\ \overline{\mathbf{V}}^{\mathrm{H}}, \tag{2}$$

where $\overline{\mathbf{D}} = \mathrm{diag}\{\lambda_1, \cdots, \lambda_S\}$, where $S \le \min\{N_R, N_T\}$, λ_s are the eigenvalues and $s \in [1, S]$ is the spatial index. According to the EVD, $\overline{\mathbf{V}}$ is the unitary eigenvector matrix of $\overline{\mathbf{H}}^{\mathrm{H}}\overline{\mathbf{H}}$. Bits and powers are allocated according to the eigenvalues λ_s. Hence, the total power is directed in the space according to the pre-processing steering matrix $\overline{\mathbf{V}}$.

Due to CSI uncertainties, a spatial equalizer is implemented to mitigate the possible inter-eigen interference [9] and compensate for the eliminated post-processing matrix (which does not diagonalize the total received channel matrix due to this CSI error). In here, we propose to use a zero-forcing equalizer as in [1]. The received signal after equalization is given by

$$\mathbf{Y} = (\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}})^{-1}\hat{\mathbf{H}}^{\mathrm{H}}\overbrace{\mathbf{H}\overline{\mathbf{V}}\mathbf{P}^{1/2}}^{\hat{\mathbf{H}}}\mathbf{X} + \tilde{\mathbf{N}} \ , \tag{3}$$

where $\hat{\mathbf{H}} = \mathbf{H}\overline{\mathbf{V}}\mathbf{P}^{1/2}$ is the aggregated channel at the receiver input, $(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}})^{-1}\hat{\mathbf{H}}^{\mathrm{H}}$ is the pseudo-inverse of it, $\mathbf{P}^{1/2}$ is the power loading matrix, and $\tilde{\mathbf{N}}$ is the colored noise at the equalizer output.

III. The QoS Sorting and Channel Adaptation

Our proposed scheme is an extension (and a modification) to the Chow-like algorithm in [1] in order to realize different user QoS assuming a different margin separation γ_u for each user u.

Without any loss of generality, the margin separations are fixed to an arbitrary step equals to Δ_γ dB. Therefore, the different users' priorities are calculated based on $\gamma_u = \gamma_{u+1} + \Delta_\gamma$ dB $\forall u \in [1, K]$. The bit-rate is given by

$$\tilde{b}_{s,k,u} = [b_{s,k,u}]_0^{b_{\max}} = \left[\log_2\left(1 + \frac{p_{s,k,u} \cdot \lambda_{s,k,u}}{\gamma_u \cdot \sigma_n^2}\right)\right]_0^{b_{\max}} \quad (4)$$

$$e^{\Delta b_{s,k,u}} = \tilde{b}_{s,k,u} - b_{s,k,u} \; , \quad (5)$$

where $\tilde{b}_{s,k,u}$ ($\in \mathbb{Z}$) is the rounding of $b_{s,k,u}$ ($\in \mathbb{R}$) to the nearest integer between $b_{\max}$ and 0, $p_{s,k,u}$ is the power allocated to the s^{th} spatial index and the k^{th} subcarrier for user u, such that $\sum_{k=1}^{N} p_k = P_T$ which is the permissible transmit power, and σ_n^2 is the white Gaussian noise variance. $e^{\Delta b_{s,k,u}}$ is the quantization error and b_{max} is the maximum allowed bits per subcarrier. Each user should be allocated with T_u bits, where the total target bit-rate is given by $B_T = \sum_{u=1}^{K} T_u$.

A. Multiuser Sorting According to the QoS

In OFDMA, any subcarrier, including all its eigenchannels, must be allocated to a single user. Therefore, the users' subcarriers have to be sorted properly in order to satisfy the QoS. In the following, we are proposing two different sorting schemes:

1) Multiuser Sorting Based on Capacity Maximization: An optimal sorting may be easily derived from the capacity equation in [7] for Gaussian channels applying water-filling. The u^{th} user maximum capacity, for N_u subcarriers ordered according to λ_s, is given by

$$\begin{aligned} \max_{U_k(N_u)} C_u &= \max_{U_k(N_u)} \frac{1}{N_u} \sum_{k=1}^{N_u} \sum_{s=1}^{S} \log_2\left(1 + \frac{p_{s,k,u}\lambda_{s,k,u}}{\sigma_n^2}\right) \quad (6) \\ &= \max_{U_k(N_u)} \frac{1}{N_u} \sum_{k=1}^{N_u} \sum_{s=1}^{S} \left(\log_2\left(\frac{\mu_u}{\sigma_n^2}\lambda_{k,s,u}\right)\right)^+ , \end{aligned}$$

where μ_u is the user u water-level constant, which is computed to meet the power constraint, $p_{s,k,u} = (\mu_u - \sigma_n^2/\lambda_{k,s,u})^+$, $U_k(N_u)$ is the set of the N_u subcarriers ordered according to λ_s, and $(a)^+ = \max(0, a)$. Thus, the maximum capacity is

$$\max_{U_k(N_u)} C_u = \max_{U_k(N_u)} \frac{1}{N_u} \sum_{k=1}^{N_u} \left(\log_2 \prod_{s=1}^{S} \left(\frac{\mu_u}{\sigma_n^2}\lambda_{k,s,u}\right)\right)^+ \quad (7)$$

Due to the monotonically increasing behavior of the logarithmic function, the right hand side is easily maximized by maximizing the product $\prod \lambda_s$. In other words, maximizing the geometric-mean of λ_s. We will refer to this method as the eigen-product sorting.

2) Multiuser Sorting Based on Rank-1 Beamforming: When the channel uncertainty increases, the stronger eigenchannels already produce a residual inter-eigen interference on each other. Therefore, another possibility could be to sort the subcarriers according to their highest eigenchannel only. Thus, the transmission power is completely directed (beamformed) towards the stronger eigenbeams. This method completely suppresses the undesired interferences from/to the weaker eigenvalues. Therefore, the error floor, which can be seen in case of residual interference, will not be considered anymore.

B. The Proposed Bit-loading

In the original algorithm [1], the noise margins are iteratively adapted to fulfill the required bit-rate using the same approximation in [4], which, however, requires many iterations to converge. Therefore, we need to compute γ_u in a more accurate way in order to place the bit-loading results closer to the correct answer. Hence, we assume that, for an odd number of users, the middle priority user ($u = m$) is equivalent to a non-prioritized single-user transmission which occupies the available N subcarriers to allocate the total target rate B_T. Thus, γ_m, under these assumed conditions, is calculated using the knowledge of N, B_T, and the signal-to-noise ratio of this user $\text{SNR}_{s,k,m}$. By neglecting the "1" in Eqn. (4) for moderate to high SNR, one can write the summation over $\bar{b}_{s,k,m}$ as

$$B_T \approx \sum_{s=1}^{S} \sum_{k=1}^{N} \log_2\left(\frac{\text{SNR}_{s,k,m}}{\gamma_m}\right) \approx SN \log_2 \frac{\overline{\text{SNR}}_m}{\gamma_m} \; , \quad (8)$$

where $\text{SNR}_{s,k,m} = \frac{p_{s,k,u} \cdot \lambda_{s,k,u}}{\gamma_u \cdot \sigma_n^2}$; let $SN = N_s$, thereby,

$$\gamma_{m_{\text{init}}} = \frac{\overline{\text{SNR}}_u}{2^{B_T/N_s}} \; , \quad (9)$$

where $\overline{\text{SNR}}$ is the average signal-to-noise ratio. The initial margin in Eqn. (9) can be used directly in the algorithm in [1], however, further fine-tuning may reduce the total number of iterations significantly. Therefore, Eqn. (9) can be rewritten without any approximation (directly derived from Eqn. (4)) as

$$\begin{aligned} B_T &= \sum_{s=1}^{S} \sum_{k=1}^{N} \log_2\left(\frac{\gamma_m + \text{SNR}_{s,k,m}}{\gamma_m}\right) \quad (10) \\ &= \sum_{s=1}^{S} \sum_{k=1}^{N} \log_2\left(\gamma_m + \text{SNR}_{s,k,m}\right) - N_s \log_2 \gamma_m \; , \end{aligned}$$

where γ_m is the only unknown in this equation. Since the addition inside the logarithm is not very sensitive to γ_m (assuming high SNRs), one can substitute directly with $\gamma_{m_{\text{init}}}$ from (9) into (11) to get a closer value of γ_m as

$$\gamma_{m_{\text{new}}} = 2^{\frac{\sum_{s=1}^{S} \sum_{k=1}^{N} \log_2\left(\gamma_{m_{\text{old}}} + \text{SNR}_{s,k,m}\right) - B_T}{N_s}} \; , \quad (11)$$

where the noise margin γ_m in Eqn. (11) is an approximated value which partially satisfies the required target bit-rate B_T assuming $b_{s,k,m} \in \mathbb{R}$. However, it is required to have integer bit values only. Therefore, γ_u has to be adapted iteratively to maintain B_T. In our case, the users data rates T_u have to be strictly fulfilled using the simple subcarrier partitioning in [1]. The following is our complete algorithm:

QoS Channel Adaptation for MIMO-OFDMA Systems

Input: $\lambda_{s,k,u}/\sigma_n^2$ of k^{th} subcarrier of the s^{th} spatial index for u^{th} user, total subcarriers N, total target bit-rate B_T, and bit-rate for every user T_u

1) The subcarriers of each user are sorted in the columns of $\mathbf{M} \in \mathbb{C}^{N \times K}$ in a descending order according to $\prod_s \lambda_s$.
2) $\gamma_{m_{\text{init}}}$ (of the middle user) is calculated as in (9). Thereafter, it is enhanced more using Eqn. (11).
3) The other noise margins are computed using this relation: $\gamma_u - \gamma_{u+1} = \Delta_\gamma$ in dB
4) $\tilde{b}_{s,k,u}$ is calculated as in (4); the number of subcarriers for each user is re-adjusted and sorted back in $\mathbf{M}$, according to T_u, using a binary search as in [2].
5) If the target bit-rate B_T is not fulfilled, γ_m is further modified using the following adjustment, as in [4] and [2],

$$\gamma_{m,new} = \gamma_{m,old} \cdot 2^{\frac{\sum_{M_u} \tilde{b}_{s,k,m} - B_T}{N}} ,$$

then go to Step 4).
6) Else, if the maximum number of iterations is achieved without fulfilling B_T, further tuning based on the quantization error, as in [4], is performed.

C. Power Allocation Scheme

The average SER within one user's transmission is assumed to be constant, while the channel gains and the bit-rates are varying. This means that the fixed power allocation cannot be suitable in this case. Even the water-filling, which has been proved to be optimum [7], does not satisfy the varying noise margins and the finite granularity of the discrete bit-loading. Therefore, the power on each subcarrier must be allocated considering the final bit-loading values, the channel eigenbeams, and the required QoS (γ_u). Finally, it is directly derived from (4) as

$$p_{s,k,u} = \gamma_u \frac{\sigma_{\text{n}}^2}{\lambda_{s,k,u}} (2^{\tilde{b}_{s,k,u}} - 1) \, . \qquad (12)$$

Apparently, the power mask will not stay perfectly constant [2] due to the previous equation. However, it will rather vary according to the number of allocated bits or the users' γ_u to have a saw-tooth like shape that fluctuates with a discontinuity at every bit-allocation change or at the user boundaries (if the bit-loading values will stay constant).

IV. Results and Discussion

To evaluate the performance of our multilevel QoS MIMO-OFDMA, we consider three users with 3 different priorities. This model should preserve an arbitrary margin separation between each two users of $\Delta_\gamma = 3$ dB. This means that a higher priority user outperforms the next user by 3 dB, for every channel condition. The number of the receive antennas N_R at the K users are assumed to be 2 (which is a reasonable assumption for a mobile terminal), while the BS provides 4 transmit antennas. This means that the resultant downlink channel is a 2×4 MIMO channel. Therefore, the OFDM transmission, which utilizes a total number of subcarriers $N = 512$, can have two eigenchannels (at most) for every subcarrier.

Figure 2 depicts the bit-loading, power allocation, and SNR for all users using the proposed optimum sorting. As can be seen in the SNR sub-plot, the first user already consumes the subcarriers with the strongest eigen-product (e.g., at the following subcarrier ranges: 0-60, 178-205, and 465-511). However, the weaker eigen-products are devoted to the weaker users (e.g., the third user consumes the left-over subchannels with the weak eigen-products, e.g., 61-174, 345-349, and 425-450). Due to the fact that the bit-loading changes and the required QoS for each user changes (with 3 dB), the power curves are rather varying by multiples of 3 dBs.

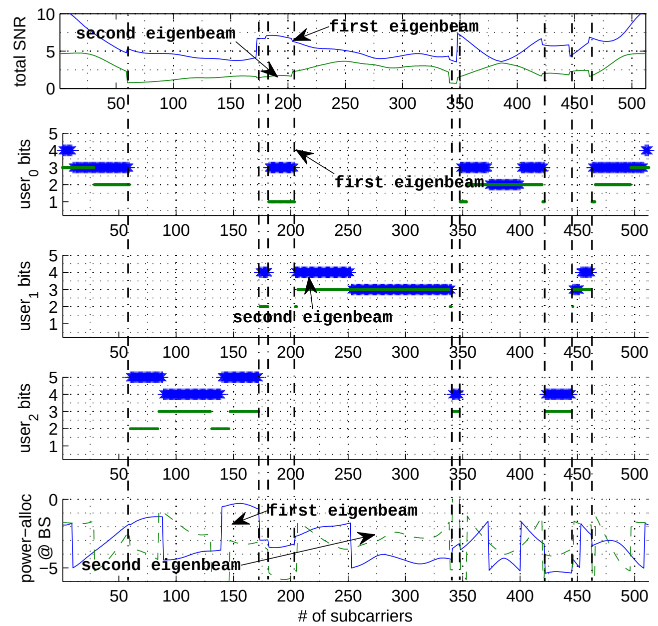

Fig. 2. Bit/power loading; SNR is in linear scale and the power is dBW

In the next part, we will examine the performance of the proposed sorting schemes for different channel conditions

A. Perfect CSI

We first assume perfect CSI conditions with the previous described channel model in Section II, and multiple users with 3 dB margin separations. In Fig. 3, the margin separation under perfect channel knowledge condition, is strictly preserved for both sorting schemes. However, the optimum sorting, that utilizes the MIMO multiplexing gain, outperforms the rank-1 eigen-beamforming by almost 3 dB. This is due to utilizing the full channel rank (double vs. single eigen-beamforming).

B. Imperfect CSI

In this step, we consider an erroneous feedback and a quantization level which delivers a non-negligible quantization error. The summation of both errors are assumed to be Gaussian (as in Sec. II) with a variance equal to 25% of the total channel variance. As seen in Fig. 4, where the 2^{nd} user's curves are removed due to space limitions, the 3 dB margin separations (or here, the 6 dB between the first and the third user) are becoming much wider with this erroneous CSI. Even

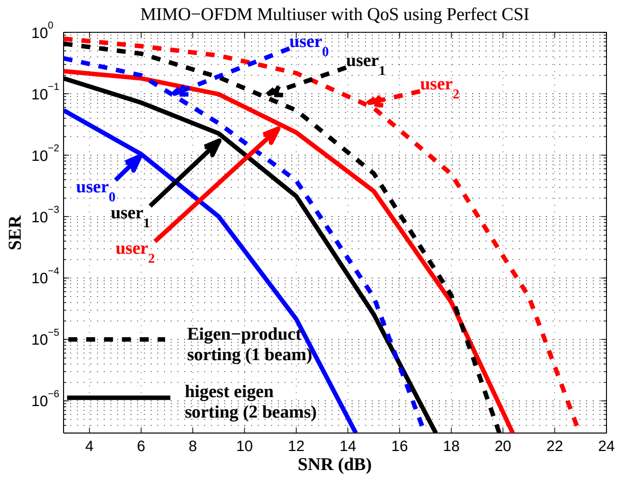

Fig. 3. Multiuser MIMO-OFDM with different QoS and different sorting schemes

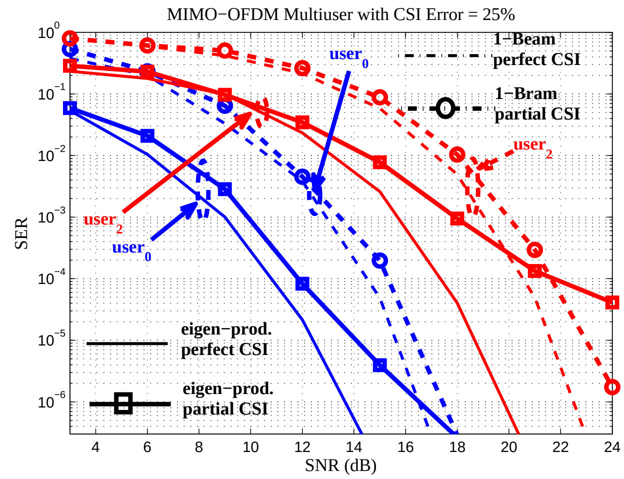

Fig. 4. Different CSI conditions for the proposed sorting schemes

more, the probability of error curves are deteriorating at high SNRs. We observe an error floor for every user, especially the lowest priority user. This is certainly due to the inter-eigen interference due to the imperfect channel diagonalization. However, the rank-1 beamforming does not suffer from this disadvantage. Thus, the performance of this sorting scheme is approaching the eigen-product sorting at high SNR. Even more, there is no error floor seen in this figure, as well, the margin separations are better preserved. This makes the latter sorting scheme very suitable for adaptive MIMO-OFDMA with CSI errors and a restricted QoS constraint. Additionally, the rank-1 sorting scheme is also faster than the optimum one.

As seen also in this figure, the performance of the rank-1 beamforming is only 1 dB worse than in case of perfect CSI case. This is due to the fact that the bit-loading is computed using erroneous CSI values, which has been accepted as a perfect CSI (at the BS). The only disadvantage of this scheme could be figured out at the lower SNRs, where the optimum sorting outperforms the rank-1 sorting by almost 2.2 dB at SER$= 10^{-5}$.

V. Conclusions

We presented a simple channel adaptation technique to realize multiuser MIMO-OFDMA with multilevel QoS. The noise margin approximation and the binary search reduces the lengthy computations and the seaching-sorting steps. The OFDMA transmission is considered to maintain the user orthogonality. However, the inter-eigen interference remains as an annoying self interference due to CSI errors. Accordingly, we presented two different sorting schemes, the eigen-product and the rank-1 (highest eigenvalue) sorting. Both schemes succeeded to preserve the margin separation (our QoS criterion) strictly in perfect CSI conditions. However, the eigen-product method starts to have a wider separation in case of partial CSI. Even more, an error floor appears at high SNRs. In contrast, the rank-1 sorting preserves the margin more strictly in this case, without any error floor.

VI. Acknowledgment

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