Lattice Coding for MIMO Systems in Impulse Noise

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***Abstract*—In this paper, we investigate signal space diversity (SSD) of lattice codes to mitigate the effect of impulse noise in wireless MIMO systems. A Middleton Class-A (MCA) model is one of the most accepted models for impulsive interference superimposed to additive white Gaussian noise (AWGN). To prove SSD for both Raleigh fading and impulse noise, we evaluate the pairwise-error probability (PEP) of optimum lattice decoding under perfect knowledge of noise states. However, due to the spatial coupling of impulse noise, the extension of the MCA model to MIMO systems leads to a correlated multivariate distribution. To maintain the full diversity advantages of lattice coding in MIMO systems, we investigate a diagonal design of lattice space-time (ST) coding for Rayleigh fading and correlated impulse noise. We also utilize a null space of the MIMO channel to extract noise states using a simple threshold detector. We show that the optimum lattice ST decoder can be realized by a noise-whitening transformation followed by a conventional sphere decoder. Finally, we evaluate the PEP of the optimum lattice ST decoding, which shows how impulse noise coupling limits the performance improvements of SSD with respect to the number of receive antennas.**

***Index Terms*—MIMO systems, lattice codes, impulse noise.**

I. INTRODUCTION

Impulsive interference corrupts a variety of practical wireless channels such as the wireless LAN spectrum at 2.4 GHz [2], [3] and digital aeronautical communications in the L-band [5]. A Middletons Class-A model [1] represents one of the most applied models for narrowband radio frequency interference (RFI). This model is confirmed [1], [6] to represent a wide class of interference varying from a pure Gaussian distribution to a heavy-tailed distribution. For multiple antenna systems, a multivariate MCA model is verified to capture the noise statistics and the spatial coupling of impulse noise [4], [7].

The research into investigating the effect of impulse noise on multiple-input multiple-output (MIMO) systems is considered in several publications [8]–[10]. One of the key advantages of the MIMO system lies in the ability of achieving both transmit and receive diversity. Signal space diversity (SSD) or so-called lattice coding [11] has been proven to provide a high diversity order for both single-input single-output (SISO) [12] and MIMO systems [13], [21]. The diversity encoder of SSD applies a unitary transform to spread the modulated symbols into an N-dimensional lattice space. The codeword of SSD can be represented as a point of a lattice, which is uniquely determined by any of the codeword components. This property allows providing a diversity of the order N in independent fading channels with AWGN. In [15], [16], Häring and Vinck introduced the concept of a complex number code (based on the inverse discrete Fourier Transform (IDFT) matrix) to mitigate the impact of impulse noise. Due to the spreading effect of the unitary transform, the optimum lattice decoding in impulse noise approaches the performance of impulse-free channels.

In this paper, we first prove the concept of SSD for wireless channels with independent fading and impulse noise. Then, we extend the analysis to a MIMO system in multivariate MCA noise. Since the extension requires perfect knowledge of noise states, we utilize the spatial dimension of MIMO channels to extract a reference signal of interference. This allows us to realize the optimum decoder as a conventional lattice decoder preceded by noise decorrelation. Thereafter, we evaluate a pairwise error probability (PEP) to assess the performance achievements of a diagonal lattice ST code in impulse noise.

The rest of this paper is organized as follows. Section II introduces the lattice code for Rayleigh fading and impulse noise. In Section III, we proceed with receiver design and performance analysis of lattice ST coding in multivariate MCA noise. Finally, simulation results and concluding remarks are presented in sections IV and V, respectively.

II. LATTICE CODES IN IMPULSE NOISE

The encoder of lattice codes applies a unitary transform or a rotation matrix to rotate an information vector $\mathbf{s}$ as [14]

$$\mathbf{x} = \mathbf{G}_N \mathbf{s}\ , \tag{1}$$

where $\mathbf{s} = [s_1, \cdots, s_N]^T$ is a vector of N complex-valued information symbols, which are taken from complex signal constellations such as QPSK or QAM. The algebraic design of $\mathbf{G}_N$ combines the $N \times N$ IDFT matrix $\mathbf{W}_N^H$ with a diagonal algebraic matrix to construct a full diversity transform as $\mathbf{G}_N = \mathbf{W}_N^H \mathrm{diag}\left(1, \theta^{\frac{1}{N}}, \cdots, \theta^{\frac{N-1}{N}}\right)$, where θ is chosen to guarantee a maximum diversity order [14]. We assume that the codeword components are transmitted through independent Rayleigh fading channels. Thus, the received signal vector can be expressed as

$$\mathbf{y} = \sqrt{E_s}\mathrm{diag}\left(\mathbf{h}^T\right)\mathbf{x} + \mathbf{z}\ , \tag{2}$$

where E_s is the transmitted energy per symbol. The vector $\mathbf{h} = [h_1, \cdots, h_N]^T$ is comprised of complex-valued random fading coefficients with unit second moment. The vector $\mathbf{z}$ represents a complex-valued additive noise at the receiver. In channels with impulse noise, the receive noise observations z_k, $k = 1, \cdots, N$, consist of Gaussian noise components $z_{G,k}$ and impulsive components $z_{I,k}$. Typically, the samples $z_{G,k}$ represent complex-valued AWGN with zero mean and variance σ_G^2. However, the impulsive components are thought to represent radio frequency interference of various man-made or natural sources [1]. An MCA model provides a sufficiently accurate representation of noise elements z_k as [1], [17]

$$p_z(z_k) = \sum_{m_k=0}^{\infty} \frac{\alpha_{m_k}}{\pi\sigma_{m_k}^2} e^{-\frac{|z_k|^2}{\sigma_{m_k}^2}} , \tag{3}$$

where

$$\sigma_{m_k}^2 = \sigma_G^2 (1 + \tfrac{m_k}{A\Upsilon}) , \tag{4}$$

and

$$\alpha_{m_k} = \frac{A^{m_k} \mathrm{e}^{-A}}{m_k!} . \tag{5}$$

The MCA model is designated using two parameters A and Υ. The former parameter $A = \lambda T_I$ is called impulsive index, where λ and T_I are the average rate (pulse per second) and the duration of impulses, respectively. The second parameter defines the Gaussian factor $\Upsilon = \sigma_G^2/\sigma_I^2$, where σ_I^2 represents the variance (average power) of the impulse component $z_{I,k}$. The MCA density reduces to a Gaussian distribution conditioned on the knoweldge of m_k, where m_k is regarded as the noise state. According to (5), the noise state m_k is a Poisson-distributed random variable such that the probability of being in a state m_k is equal to α_{m_k}. Asymptotically, the MCA model approaches a Gaussian distribution with zero mean and variance $\sigma^2 = \sigma_G^2(1 + \frac{1}{\Upsilon})$ when $A \to \infty$. In cases when $A < 1$, the MCA interference exhibits an impulsive appearance. The MCA model can be approximated by a 2-term Gaussian mixture (GM) model as [18]

$$p_z(z_k) = \frac{\alpha_0}{\pi\sigma_0^2} e^{-\frac{|z_k|^2}{\sigma_0^2}} + \frac{\alpha_1}{\pi\sigma_1^2} e^{-\frac{|z_k|^2}{\sigma_1^2}} , \tag{6}$$

where $\alpha_0 = 1 - A$ and $\alpha_1 = A$. In (6), we note that the first term $m_k = 0$ and the second term $m_k = 1$ are corresponding to the Gaussian and impulsive states of noise with probability of occurrence $1 - A$ and A, respectively. For this reason, the impulsive index A is recognized as the duty cycle of impulses [17].

A. Signal Space Diversity

Similar to [12], we assume that the channel gains h_k, $k = 1, \cdots, N$, are known at the receiver. Indeed, the independence assumption for fading coefficients represents the situation where the code components x_k are interleaved in time. This assumption also implies that the noise observations z_k are independent. To prove SSD of lattice codes in impulse noise, we evaluate the pairwise error probability (PEP) of the optimum lattice decoding. Since the exact evaluation of the PEP is infeasible [15], we further assume that the noise states m_k, $k = 1, \cdots, N$, are known at the receiver. Hence, the distribution of $\mathbf{z}$ reduces to conditional Gaussian as

$$p_{\mathbf{z}}(\mathbf{z}|\mathbf{m}) = \prod_{k=1}^{N} \frac{1}{\pi\sigma_{m_k}^2} e^{-\frac{|z_k|^2}{\sigma_{m_k}^2}} , \tag{7}$$

where $\mathbf{m} = [m_1, \cdots, m_N]^T$ is a noise state vector. We assume that the detector decides between two lattice codeword $\mathbf{x}_i$ and $\mathbf{x}_j$, $\forall i \neq j$. Therefore, the optimum decision rule for deciding between $\mathbf{x}_i$ and $\mathbf{x}_j$ can be derived as

$$\log\left(\frac{p_{\mathbf{z}}(\mathbf{y} - \sqrt{E_s}\mathbf{h}\cdot\mathbf{x}_i|\mathbf{h},\mathbf{m})}{p_{\mathbf{z}}(\mathbf{y} - \sqrt{E_s}\mathbf{h}\cdot\mathbf{x}_j|\mathbf{h},\mathbf{m})}\right) \underset{\mathbf{x}_j}{\overset{\mathbf{x}_i}{\gtrless}} 0 . \tag{8}$$

Substituting (7) into (8), the decision rule in (8) leads to

$$2\sqrt{E_s}\sum_{k=1}^{N}\mathrm{Re}\{\frac{h_k^*}{\sigma_{m_k}^2}(x_{i,k}^* - x_{j,k}^*)y_k\} - E_s\sum_{k=1}^{N}\frac{|h_k|^2}{\sigma_{m_k}^2}(|x_{i,k}|^2 - |x_{j,k}|^2) \underset{\mathbf{x}_j}{\overset{\mathbf{x}_i}{\gtrless}} 0 . \tag{9}$$

We suppose that $\mathbf{x}_i$ was sent, i.e., $y_k = \sqrt{E_s}h_k x_{i,k} + z_k$. Hence, the decision variable in (9) reduces to

$$\chi = 2\sqrt{E_s}\sum_{k=1}^{N}\mathrm{Re}\{\frac{h_k^*}{\sigma_{m_k}^2}(x_{i,k}^* - x_{j,k}^*)z_k\} + E_s\sum_{k=1}^{N}\frac{|h_k|^2}{\sigma_{m_k}^2}|x_{i,k} - x_{j,k}|^2 , \tag{10}$$

The PEP is simply the probability of erroneously decoding $\mathbf{x}_j$ given that $\mathbf{x}_i$ was transmitted, which can be evaluated as the probability that χ is less than zero. The decision variable χ is Gaussian with mean $\mu_\chi = E_s\sum_{k=1}^{N}\frac{|h_k|^2}{\sigma_{m_k}^2}(|x_{i,k} - x_{j,k}|^2)$ and variance $\sigma_\chi^2 = 2E_s\sum_{k=1}^{N}\frac{|h_k|^2}{\sigma_{m_k}^2}|x_{i,k} - x_{j,k}|^2$. Hence, the conditional PEP can be computed as

$$P(\mathbf{x}_i \to \mathbf{x}_j|\mathbf{h},\mathbf{m}) = Q\left(\frac{\mu_\chi}{\sigma_\chi}\right) , \tag{11}$$
$$= Q\left(\sqrt{\frac{E_s}{2}\sum_{k=1}^{N}\frac{|h_k|^2}{\sigma_{m_k}^2}|x_{i,k} - x_{j,k}|^2}\right) ,$$

where $Q(x) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$. Using the Chernoff bound, (11) can be upper-bounded as

$$P(\mathbf{x}_i \to \mathbf{x}_j|\mathbf{h},\mathbf{m}) \leq \frac{1}{2}\prod_{k=1}^{N}\exp\left(-\frac{E_s}{4}\frac{|h_k|^2}{\sigma_{m_k}^2}|x_{i,k} - x_{j,k}|^2\right) . \tag{12}$$

Since h_k are complex-valued Gaussian distributed random variables, $\beta_k = |h_k|^2$ follows a chi-square distribution with two degrees of freedom. We evaluate the conditional PEP in (12) over the distribution of β_k as follows:

$$P(\mathbf{x}_i \to \mathbf{x}_i|\mathbf{m}) = \frac{1}{2}\prod_{k=1}^{N}\int_0^\infty e^{-\frac{E_s}{4}\frac{\beta_k}{\sigma_{m_k}^2}|x_{i,k}-x_{j,k}|^2} p(\beta_k)d\beta_k ,$$
$$\leq \frac{1}{2}\prod_{k=1}^{N}\frac{1}{1 + \frac{E_s}{4}\frac{|x_{i,k}-x_{j,k}|^2}{\sigma_{m_k}^2}} . \tag{13}$$

Since $\sigma^2_{m_k} = \sigma^2_G(1+\frac{m_k}{A\Upsilon})$, $k = 1, \cdots, N$, the right-hand side of (13) can be approximated at a high signal-to-noise ratio (SNR) E_s/σ^2_G as

$$P(\mathbf{x}_i \to \mathbf{x}_j|\mathbf{m}) < \tfrac{1}{2}\left(\frac{E_s}{4\sigma^2_G}\right)^{-N} \prod_{k=1}^{N} \frac{1+\frac{m_k}{A\Upsilon}}{|x_{i,k}-x_{j,k}|^2} \ . \quad (14)$$

It is worth mentioning that in an impulse-free case, i.e., $m_k = 0$, $\forall k$, (14) can be written as

$$P(\mathbf{x}_i \to \mathbf{x}_j|\mathbf{m}) < \tfrac{1}{2}\left(\frac{E_s}{4\sigma^2_G}\right)^{-N} \frac{1}{d_p^{(N)}(\mathbf{x}_i, \mathbf{x}_j)} \ , \quad (15)$$

where $d_p^{(N)}(\mathbf{x}_i, \mathbf{x}_j) = \prod_{k=1}^{N} |x_{i,k} - x_{j,k}|^2$ is the N-product distance [11], [12] between $\mathbf{x}_i$ and $\mathbf{x}_j$. To derive a closed-form expression of the PEP for channels with impulse noise, we have to average (14) over the statistics of m_k, $k = 1, \cdots, N$. Thus, we obtain

$$P(\mathbf{x}_i \to \mathbf{x}_j) < \overbrace{\tfrac{1}{2}\left(\frac{E_s}{4\sigma^2_G}\right)^{-N}}^{g_d(N)} \overbrace{\frac{\mathrm{E}_{\mathbf{m}}\{\prod_{k=1}^{N}(1+\frac{m_k}{A\Upsilon})\}}{d_p^{(N)}(\mathbf{x}_i,\mathbf{x}_j)}}^{g_c(\mathbf{x}_i,\mathbf{x}_j)} \ , \quad (16)$$

where $\mathrm{E}_{\mathbf{m}}\{\cdot\}$ denotes the expectation with respect to a noise state vector $\mathbf{m} = [m_1, \cdots, m_N]$. The PEP expression in (16) allows us to distinguish between diversity gain $g_d(N)$ and coding gain $g_c(\mathbf{x}_i, \mathbf{x}_j)$ of the optimum lattice decoder in impulse noise. In (16), we observe that the lattice code maintains a maximum diversity order N. Additionally, we note that the coding gain is depending on noise states probabilities. In a 2-term GM model, the noise states probabilities are $\alpha_0 = 1 - A$ and $\alpha_1 = A$ such that $\mathrm{E}_{m_k}\{m_k\} = A$. The received noise observations z_k are assumed to be statistically independent, this implies that the noise states m_k, $\forall k$, are also independent. Thus, the coding gain $g_c(\mathbf{x}_i, \mathbf{x}_j)$ can be evaluated as

$$g_c(\mathbf{x}_i, \mathbf{x}_j) = \frac{\prod_{k=1}^{N}\left(1+\frac{\mathrm{E}_{m_k}\{m_k\}}{A\Upsilon}\right)}{d_p^{(N)}(\mathbf{x}_i,\mathbf{x}_j)} ,= \frac{(1+\frac{1}{\Upsilon})^N}{d_p^{(N)}(\mathbf{x}_i,\mathbf{x}_j)} \ , \quad (17)$$

where the factor $(1+\frac{1}{\Upsilon})^N$ represents the performance loss due to the presence of impulse noise. Substituting (17) into (16) yields

$$\begin{aligned} P(\mathbf{x}_i \to \mathbf{x}_j) &< \tfrac{1}{2}\left(\frac{E_s}{4\sigma^2_G}\right)^{-N} \frac{(1+\frac{1}{\Upsilon})^N}{d_p^{(N)}(\mathbf{x}_i, \mathbf{x}_j)} \ , \\ &< \tfrac{1}{2}\left(\frac{E_s}{4\sigma^2_G(1+\frac{1}{\Upsilon})}\right)^{-N} \frac{1}{d_p^{(N)}(\mathbf{x}_i, \mathbf{x}_j)} \ , \end{aligned} \quad (18)$$

where the factor $(1+\frac{1}{\Upsilon})$ determines the gap in the SNR between the performances of the optimum lattice decoders for impulse noise and the impulse-free case.

B. Extension to MIMO systems

We consider a point-to-point MIMO system with N_T transmit and N_R receive antennas. Earlier works on ST coding consider an orthogonal space-time block code (OSTBC) to mitigate the impact of impulse noise [8], [9]. Hereto, we employ a lattice space-time (ST) coding scheme to generate a square ST code matrix of size $N_T \times N_K$ such that N_T is equal to the number of time slots N_K as

$$\mathbf{X} = \begin{pmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,N_T} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_T,1} & x_{N_T,2} & \cdots & x_{N_T,N_T} \end{pmatrix} , \quad (19)$$

where the entries $x_{n_T,k}$ denote the coded symbols transmitted from the n_T^{th} transmit antenna at a time slot k. The MIMO channel is assumed to be i.i.d. complex Gaussian with quasi-static flat fading, i.e., the channel remains constant during the transmission time of the lattice code matrix. The existing lattice ST code matrices utilize the space and time dimensions of the MIMO channel to provide both spatial multiplexing (SM) and full SSD [19], [20]. The received signal vector $\mathbf{y}_k = [y_{1,k}, \cdots, y_{N_R,k}]^T$ during the k^{th} time slot can be expressed as

$$\mathbf{y}_k = \sqrt{E_s}\mathbf{H}\mathbf{x}_k + \mathbf{z}_k \ , \quad k = 1, \cdots, N_T \ , \quad (20)$$

where $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ is the MIMO channel matrix and $\mathbf{x}_k = [x_{1,k}, \cdots, x_{N_T,k}]^T$ represents the k^{th} column vector of $\mathbf{X}$. Here $\mathbf{z}_k = [z_{1,k}, \cdots, z_{N_R,k}]^T$ is a received spatial noise vector. In strong interference channels, the N_R receive antennas are typically affected by impulse noise generated by the same ISM sources [4], [7]. Thus, a multivariate MCA model can be used to represent the joint probability as

$$p(\mathbf{z}_k) = \tfrac{1-A}{\pi^{N_R}|\mathbf{\Sigma}_0|} e^{-\mathbf{z}_k^H \mathbf{\Sigma}_0^{-1} \mathbf{z}_k} + \tfrac{A}{\pi^{N_R}|\mathbf{\Sigma}_1|} e^{-\mathbf{z}_k^H \mathbf{\Sigma}_1^{-1} \mathbf{z}_k} \ , \quad (21)$$

The spatial coupling of impulse noise is described by the covariance matrix $\mathbf{\Sigma}_{m_k}$, $m_k = 0, 1$, as

$$\mathbf{\Sigma}_{m_k} = \begin{pmatrix} \sigma^2_{m_k,1} & \cdots & \rho^{m_k}_{1N_R}\sigma_{m_k,1}\sigma_{m_k,N_R} \\ \vdots & \ddots & \vdots \\ \rho^{m_k}_{N_R 1}\sigma_{m_k,N_R}\sigma_{m_k,1} & \cdots & \sigma^2_{m_k,N_R} \end{pmatrix} , \quad (22)$$

where $\sigma^2_{m_k,n_R} = \sigma^2_G(1+\frac{m_k}{A\Upsilon_{n_R}})$ denotes the noise variance at the n_R^{th} receive antenna and $\rho^{m_k}_{n_R \grave{n}_R}$, $m_k = 0, 1$, are the correlation coefficients between the n_R^{th} and $\grave{n}_R^{\text{th}}$ receive antennas of the Gaussian and impulsive components, respectively. In impulse-free channels, $m_k = 0$, the noise observations are uncorrelated, i.e., $\mathbf{\Sigma}_0 = \sigma^2_G\mathbf{I}_{N_R}$. However, in the channels with impulse noise, $m_k = 1$, the received impulse noise observations are dependent and might be correlated such as $\mathbf{\Sigma}_1$ is positive-semidefinite and symmetric. Due to the spatial coupling of impulse noise, it is more reasonable to design a lattice ST code matrix that guarantees a maximum diversity order $N_T N_R$. In such cases [19], the lattice code matrix entries $x_{n_T,k}$ should examine independent observations for both fading and impulse noise. This directs our attention to investigate a diagonal lattice ST code to mitigate the impact of impulse noise. The diagonal lattice ST code utilizes only the main diagonal elements of $\mathbf{X}$ to interleave the code components $\mathbf{x} = \mathbf{G}_{N_T}\mathbf{s}$ as

$$\mathbf{X} = \mathrm{diag}\left(\mathbf{x}^T\right) = \begin{pmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & x_{N_T} \end{pmatrix} . \quad (23)$$

This code proved to achieve maximum diversity and support a rate of 1 symbol per channel use for fading channels with AWGN. In the following analysis, we restrict our analysis to the diagonal ST code in impulse noise. However, the generalization to an arbitrary lattice ST coding scheme is straightforward.

III. Receiver Design and Performance Analysis

The previous analysis assumes that the noise states m_k, $k = 1, \cdots, N_T$, are known at the receiver. In this section, we realize this assumption using the null space of MIMO channels at the receiver. Hence, we proceed with the performance evaluation of lattice ST coding in impulse noise.

A. Decoding of Lattice ST Codes

The practical realization of the lattice decoding for MIMO systems requires perfect knoweldge of noise states. To provide an efficient noise state estimate, the properties of impulse noise along time, space, and frequency dimensions can be used to provide a reference signal of impulse noise. Herein, we take a step into involving the spatial coupling of impulse noise to extract the reference signal via the null space. We consider that the receiver is equipped with $N_R = N_T + 1$ antennas. Using a QR decomposition, we factorize the channel $\mathbf{H}$ into a product of an $N_T + 1 \times N_T + 1$ unitary matrix $\mathbf{Q}$ and an $N_T + 1 \times N_T$ upper triangular matrix $\mathbf{R} = \begin{pmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{pmatrix}$, where the last row of $\mathbf{R}$ consists entirely of zeros. Thus, we multiply $\mathbf{Q}^H$ by the received signal vector $\mathbf{y}_k = \sqrt{E_s}\mathbf{H}\mathbf{x}_k + \mathbf{z}_k$, which yields

$$\tilde{\mathbf{y}}_k = \sqrt{E_s}\begin{pmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{pmatrix}\mathbf{x}_k + \mathbf{Q}^H\mathbf{z}_k \ , \ k = 1, \cdots, N_T \ , \quad (24)$$

where the bottom element of $\tilde{\mathbf{y}}_k$ contains only a noise component as

$$\tilde{y}_{N_R,k} = \sum_{n_R=1}^{N_T+1} q^*_{n_R,N_R} z_{n_R,k} \ , \quad k = 1, \cdots, N_T \, . \quad (25)$$

For $m_k = 0$, the noise observations $z_{n_R,k}$, $n_R = 1, \cdots, N_T + 1$, are Gaussian with $\mathbf{\Sigma}_0 = \sigma_G^2 \mathbf{I}_{N_R}$. Hence, the reference signal $\tilde{y}_{N_R,k}$ is still Gaussian with zero mean and variance σ_G^2. For $m_k = 1$, since $z_{n_R,k}$, $n_R = 1, \cdots, N_T + 1$, are spatially coupled and correlated, $\tilde{y}_{N_R,k}$ provides a constructive signal of impulse noise. For mathematical convenience, we consider uncorrelated MCA spatial observations, i.e., $\rho_{n_R \grave{n}_R} = 0$, $\forall n_R \neq \grave{n}_R$. Hence, the reference signal $\tilde{y}_{N_R,k}$ follows a 2-term GM distribution with variance $\tilde{\sigma}^2_{N_R,m_k} = \sigma_G^2 \left(1 + \frac{m_k}{A}\sum_{n_R=1}^{N_T+1} \frac{|q_{n_R,N_R}|^2}{\Upsilon_{n_R}}\right)$. Hence, the noise states m_k can be detected as

$$m_k = \begin{cases} 1 & \text{if } |\tilde{y}_{N_R,k}|^2 \geq c_0^2 \\ 0 & \text{otherwise} \end{cases} , \quad k = 1, \cdots, N_T \, , \quad (26)$$

where $c_0 = \sqrt{\frac{\tilde{\sigma}^2_{N_R,0}\tilde{\sigma}^2_{N_R,1}}{\tilde{\sigma}^2_{N_R,1} - \tilde{\sigma}^2_{N_R,0}} \log \frac{(1-A)\tilde{\sigma}^2_{N_R,1}}{A\tilde{\sigma}^2_{N_R,0}}}$ denotes an impulse detection threshold [18]. By arranging the received vectors $\mathbf{y}_k = \sqrt{E_s}\mathbf{H}\mathbf{x}_k + \mathbf{z}_k$, $k = 1, \cdots, N_T$, into a single column vector $\mathbf{y} = \left(\mathbf{y}_1^T, \cdots, \mathbf{y}_{N_T}^T\right)^T$ as

$$\mathbf{y} = \sqrt{E_s} \overbrace{\begin{pmatrix} \mathbf{h}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_{N_T} \end{pmatrix}}^{\mathbf{H}_U} \mathbf{x} + \mathbf{z} \ , \quad (27)$$

where $\mathbf{h}_k = [h_{1k}, \cdots, h_{N_R k}]^T$ is the k^{th} column vector of a MIMO channel $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$. Thus, $\mathbf{H}_U \in \mathbb{C}^{N_R N_T \times N_T}$ can seen a united MIMO channel matrix [19]. Since the noise state estimates m_k, $k = 1, \cdots, N_T$, are extracted from a null space, the noise vector $\mathbf{z} = \left[\mathbf{z}_1^T, \cdots, \mathbf{z}_{N_T}^T\right]^T$ is a multivariate Gaussian vector with

$$\mathbf{\Sigma}_{\mathbf{m}} = \mathrm{E}\left[\mathbf{z}\mathbf{z}^H\right] = \begin{pmatrix} \mathbf{\Sigma}_{m_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_{m_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Sigma}_{m_{N_T}} \end{pmatrix} , \quad (28)$$

where $\mathbf{\Sigma}_{m_k}$ is the spatial covariance matrix (22) of the k^{th} time slot noise observations. Since impulse noise is spatially coupled, a noise-whitening matrix is applied to obtain the equivalent samples with uncorrelated noise. The inverse of the covariance matrix $\mathbf{\Sigma}_{\mathbf{m}}^{-1}$ can be factorized as $\mathbf{\Sigma}_{\mathbf{m}}^{-1} = \mathbf{L}\mathbf{L}^H$, multiplying (27) by $\mathbf{L}^H$, we obtain $\dot{\mathbf{y}} = \mathbf{L}^H\mathbf{y}$ and $\dot{\mathbf{z}} = \mathbf{L}^H\mathbf{z}$. The elements of $\dot{\mathbf{z}}$ are i.i.d. Gaussian distributed random variables with unit variance. Hence, the ML decoder can be implemented as

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s} \in \mathbb{C}^{N_T}} \left|\dot{\mathbf{y}} - \sqrt{E_s}\mathbf{L}^H\mathbf{H}_U\mathbf{G}_{N_T}\mathbf{s}\right|^2 , \quad (29)$$

which can be realized using a conventional sphere decoder.

B. Pairwise Error Probability

To assess the PEP of the diagonal lattice ST code, we rewrite the received signal vector (27) in terms of a diagonal lattice ST code matrix $\mathbf{X} = \mathrm{diag}(\mathbf{x}^T)$ as

$$\mathbf{y} = \sqrt{E_s}\left(\mathbf{X}^T \otimes \mathbf{I}_{N_R}\right)\mathbf{h} + \mathbf{z} \ , \quad (30)$$

where $\otimes$ is the Kronecker product, $\mathbf{I}_{N_R}$ denotes the identity matrix of size N_R, and $\mathbf{h} = \left(\mathbf{h}_1^T, \cdots, \mathbf{h}_{N_T}^T\right)^T$ sorts the MIMO channel $\mathbf{H}$ into a single column vector. We assume that the receiver can decide between two lattice code matrices $\mathbf{X}_i$ and $\mathbf{X}_j$. The probability that $\mathbf{X}_i$ was sent and $\mathbf{X}_j$ is detected can be expressed as

$$P(\mathbf{X}_i \to \mathbf{X}_j | \mathbf{h}, \mathbf{m}) = Q\left(\sqrt{\frac{E_s}{2N_T}\mathbf{h}^H\mathbf{B}\mathbf{h}}\right) , \quad (31)$$

where $\mathbf{B} = (\mathbf{\Psi}_i - \mathbf{\Psi}_j)^H \mathbf{\Sigma}_{\mathbf{m}}^{-1} (\mathbf{\Psi}_i - \mathbf{\Psi}_j)$ is a code difference matrix with $\mathbf{\Psi} = \mathbf{X}^T \otimes \mathbf{I}_{N_R}$. For a Hermitian matrix $\mathbf{B}$, the eigen-decomposition implies that $\mathbf{B} = \mathbf{V}\mathbf{\Delta}\mathbf{V}^H$, where

$\mathbf{V}$ is a unitary matrix and $\boldsymbol{\Delta}$ is an $N_T N_R \times N_T N_R$ diagonal eigenvalues matrix. Substituting this into (31) yields

$$P(\mathbf{X}_i \to \mathbf{X}_j|\mathbf{h},\mathbf{m}) = Q\left(\sqrt{\tfrac{E_s}{2}\boldsymbol{\hbar}^H \boldsymbol{\Delta}\boldsymbol{\hbar}}\right) , \quad (32)$$
$$= Q\left(\sqrt{\tfrac{E_s}{2}\sum_{n_T=1}^{N_T}\sum_{n_R=1}^{N_R}|\hbar_{n_R n_T}|^2 \lambda_{(n_T-1)N_R+n_R}}\right) ,$$

where $\boldsymbol{\hbar} = \mathbf{V}^H \mathbf{h}$ and $\lambda_{\lambda_{(n_T-1)N_R+n_R}}$, $1 \leq n_T \leq N_T$ and $1 \leq n_R \leq N_R$, are the eigenvalues of $\mathbf{B}$. Since $\mathbf{V}$ is unitary, then $\boldsymbol{\hbar}$ follows the same distribution of $\mathbf{h}$. Therefore, $\beta_{n_R n_T} = |\hbar_{n_R n_T}|^2, \forall n_R$ and $\forall n_T$, are i.i.d. chi-square variables. Similar to (12) and (13), we apply the Chernoff bound and average the right-hand side with respect the statistics of $\beta_{n_R n_T}$ to arrive at

$$P(\mathbf{X}_i \to \mathbf{X}_j|\mathbf{m}) \leq \tfrac{1}{2}\prod_{n_T=1}^{N_T}\prod_{n_R=1}^{N_R}\frac{1}{1+\frac{E_s}{4}\lambda_{(n_T-1)N_R+n_R}} . \quad (33)$$

At high SNRs, the right-hand side of (33) can be approximated as

$$P(\mathbf{X}_i \to \mathbf{X}_j|\mathbf{m}) < \tfrac{1}{2}\left(\tfrac{E_s}{4}\right)^{-N_T N_R}\prod_{n_T=1}^{N_T}\prod_{n_R=1}^{N_R}\frac{1}{\lambda_{(n_T-1)N_R+n_R}} . \quad (34)$$

Since $\mathbf{X}$ is a diagonal matrix, the eigenvalues of $\mathbf{B}$ can be expressed as [9]

$$\lambda_{(n_T-1)N_R+n_R} = |x_{i,n_T} - x_{j,n_T}|^2 \zeta_{n_R,n_T} \ , n_R = 1,\cdots,N_R \ , \quad (35)$$

where $\zeta_{n_R,k}$, $n_R = 1, \cdots, N_R$, correspond to the eigenvalues of $\boldsymbol{\Sigma}_{m_k}^{-1}$ for spatial noise observations $\mathbf{z}_k$ at the k^{th} time slot. Substituting (35) into (34) yields

$$P(\mathbf{X}_i \to \mathbf{X}_j|\mathbf{m}) < \tfrac{1}{2}\left(\tfrac{E_s}{4}\right)^{-N_T N_R}\frac{\prod_{k=1}^{N_T}\prod_{n_R=1}^{N_R}\frac{1}{\zeta_{n_R,k}}}{\left(d_p^{(N_T)}(\mathbf{x}_i,\mathbf{x}_j)\right)^{N_R}} . \quad (36)$$

For correlated impulse noise, one should compute the eigenvalues of $\boldsymbol{\Sigma}_{m_k}^{-1}$. For simplicity, we consider uncorrelated MCA observations, i.e., $\boldsymbol{\Sigma}_{m_k} = \text{diag}(\sigma_{1,m_k}^2, \cdots, \sigma_{N_R,m_k}^2)$. Therefore, the eigenvalues of $\boldsymbol{\Sigma}_{m_k}^{-1}$ can be expressed as

$$\zeta_{n_R,k} = \frac{1}{\sigma_G^2\left(1+\frac{m_k}{A\Upsilon_{n_R}}\right)} , \quad n_R = 1,\cdots,N_R . \quad (37)$$

Substituting this into (36) yields

$$P(\mathbf{X}_i \to \mathbf{X}_j|\mathbf{m}) < \tfrac{1}{2}\left(\tfrac{E_s}{4\sigma_G^2}\right)^{-N_R N_T}\frac{\prod_{k=1}^{N_T}\prod_{n_R=1}^{N_R}\left(1+\frac{m_k}{A\Upsilon_{n_R}}\right)}{\left(d_p^{(N_T)}(\mathbf{x}_i,\mathbf{x}_j)\right)^{N_R}} . \quad (38)$$

Substituting $m_k = 0$ into (38) yields the PEP for impulse-free channels as

$$P(\mathbf{X}_i \to \mathbf{X}_j) < \tfrac{1}{2}\left(\tfrac{E_s}{4\sigma_G^2}\right)^{-N_R N_T}\frac{1}{\left(d_p^{(N_T)}(\mathbf{x}_i,\mathbf{x}_j)\right)^{N_R}} . \quad (39)$$

Similar to (16), we average (38) with respect to statistics of noise states m_k, $k = 1, \cdots, N_T$, as

$$P(\mathbf{X}_i \to \mathbf{X}_j) < \overbrace{\tfrac{1}{2}\left(\tfrac{E_s}{4\sigma_G^2}\right)^{-N_T N_R}}^{g_d(N_T N_R)} \times \overbrace{\frac{\left(\mathrm{E}_{m_k}\left\{\prod_{n_R=1}^{N_R}\left(1+\frac{m_k}{A\Upsilon_{n_R}}\right)\right\}\right)^{N_T}}{\left(d_p^{(N_T)}(\mathbf{x}_i,\mathbf{x}_j)\right)^{N_R}}}^{g_c(\mathbf{x}_i,\mathbf{x}_j)} , \quad (40)$$

which proves the maximum diversity order $N_T N_R$ of the diagonal lattice ST codes for MIMO systems in impulse noise. However, to investigate the coding gain $g_c(\mathbf{x}_i, \mathbf{x}_j)$, we simply consider MCA noise with $\Upsilon_{n_R} = \Upsilon$, $n_R = 1, \cdots, N_R$. Therefore, since $\alpha_0 = 1 - A$ and $\alpha_1 = A$, the coding gain can be expressed as

$$g_c(\mathbf{x}_i,\mathbf{x}_j) = \frac{\left(1-A+A\left(1+\frac{1}{A\Upsilon}\right)^{N_R}\right)^{N_T}}{\left(d_p^{(N_T)}(\mathbf{x}_i,\mathbf{x}_j)\right)^{N_R}} . \quad (41)$$

Comparing (41) and (18), we can see how the number of receive antennas N_R affects the performance of lattice codes in impulse noise. For $N_R = 1$, we observe that the coding gain of the diagonal lattice ST code (41) approaches the coding gain of SSD in impulse noise (18). However, for $N_R > 1$, we see that the code gain is further limited by the impulsive index A of MCA noise. This limitation increases as the number of receive antennas increases.

IV. Simulation Results

We performed a link level simulations to assess the performance improvement of lattice coding for wireless communications in impulse noise. First, we confirm the concept of SSD in impulse noise. Thus, a QPSK vector is precoded using a unitary transform $\mathbf{G}_N$ with $\theta = e^{j\pi/6}$ [20]. The lattice codeword is transmitted on N independent channels for both Rayleigh fading and impulse noise. Figure 1 illustrates the bit-error ratio (BER) versus E_s/σ_G^2 for lattice codes with $N = 1$, 2, and 4 in MCA noise with $A = 0.1$ and $\Upsilon = 0.01$. Additionally, we depict the upper performance bound on the PEP and the impulse-free case as references. We observe that the performance of the lattice code improves as N increases, which agrees with the diversity order of SSD. We also note that the gap to the impulse-free limit is consistent with the coding gain factor $(1 + \frac{1}{\Upsilon})$. Second, we simulated the BER performance of the lattice ST decoder for MIMO systems based on null space estimates of noise states. Figure 2 depicts the BER results of the 2×3 MIMO system in spatially uncorrelated MCA noise with two different A and $\Upsilon = 0.01$. We note that, at high SNRs, the gap in the SNR between the performances of lattice ST coding in impulse noise and an impulse-free case increases as the impulsive index A decreases. This gap is consistent with the performance loss of the lattice ST decoding in (41).

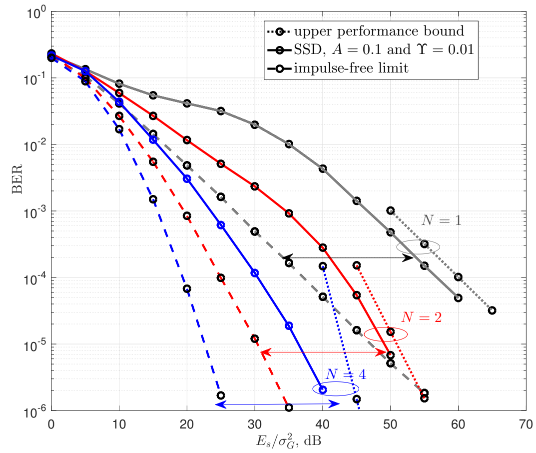

Fig. 1. BER performances of the lattice code in impulse noise with $A = 0.1$ and $\Upsilon = 0.01$ for different N

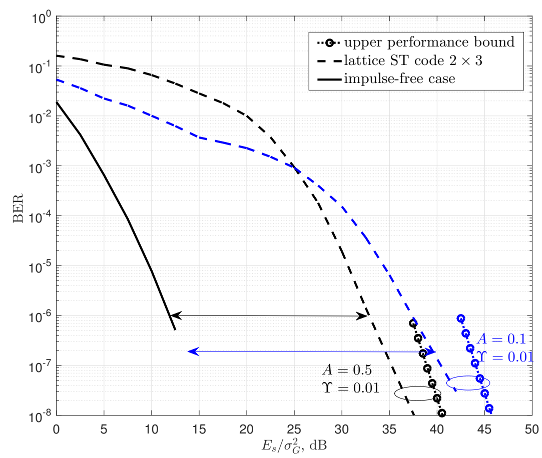

Fig. 2. BER Performance of the lattice ST code for a 2×3 MIMO system in impulse noise

V. Conclusion

In this paper, we have investigated signal space diversity based codes for multiple-input multiple-output systems in fading channels with spatially coupled impulsive interference. We adopt an MCA model to represent the amplitude statistics of received interference superimposed to background Gaussian noise. This model is well defined using two parameters to fit a wide class of interference distributions. For MIMO systems, we use a multivariate MCA model to capture the spatial dependency and correlation of impulse noise at the different receive antennas. First, we evaluated the pairwise error probability of SSD to investigate the performance gain of lattice codes in independent Rayleigh fading and impulse noise. We showed that the coding gain is limited by the Gaussian factor of impulse noise. Then, the PEP analysis is extended to a lattice space-time code of MIMO systems in spatially coupled impulse noise channels. The PEP showed that the performance gain of the lattice ST code is limited by the impulsive index of noise. In addition, this limitation grows exponentially with the number of receive antennas. Since the analysis requires perfect knowledge of noise states, we utilize a null space of a MIMO channel to estimate the state of impulse noise. Finally, we presented simulation results showing the performance comparisons for lattice codes in different impulse noise.

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