Sorting Bits into RS Symbols According to their Reliability

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Abstract — **This paper discusses the advantages that can be expected from sorting bits into groups with similar reliability and gathering them into the symbols of a Reed-Solomon code.**

I. Motivation

In ADSL, a so-called 'tone ordering' has been provided to route bits from high-SNR tones through the interleaved channel, since those are more vulnerable to unexpected non-stationary noise. Gathering bits with equal reliability into common RS symbols would be a further step.

II. Results with two different bit error probabilities

We assume two different bit error rates p_i, $i = 1,2$ and compare the resulting bit error rates after the decoding of the RS code with sorting with the error rate obtained without sorting.

The symbol-error rates of RS symbols of length m are

$$p_{si} = 1 - (1-p_i)^m \quad \text{or} \quad p_{sm} = 1 - \prod_{i=1}^{2} (1-p_i)^{m/2}\,, \tag{1}$$

if the symbols are either filled with bits ordered according to their reliability or equally filled with bits of all reliability classes, respectively.

On the basis of these symbol error probabilities, we approximate the bit error probabilities after the decoding as

$$p_{bm,dec} \approx \sum_{j=t+1}^{N} \frac{j}{N} \cdot \frac{p_{bm}}{p_{sm}} \cdot \binom{N}{j} p_{sm}^{j} (1-p_{sm})^{N-j}\,, \tag{2}$$

$$p_{b,dec} \approx \sum_{j=t+1}^{N} \sum_{j_1=0}^{\min(j,N/2)} \binom{N/2}{j_1} p_{s1}^{j_1} (1-p_{s1})^{N/2-j_1} \cdot$$
$$\cdot \binom{N/2}{j-j_1} p_{s2}^{j-j_1} (1-p_{s2})^{N/2-j+j_1} \left(j_1 \frac{p_{b1}}{p_{s1}} + (j-j_1)\frac{p_{b2}}{p_{s2}} \right) / N\,, \tag{3}$$

where we assume that the symbols of uncorrected received words have the same average bit error probability as the ones before decoding.

Figure 1 shows the ratio $p_{b,dec}/p_{bm,dec}$ with two different bit error rates p_1 and p_2. Although the number of bits was chosen to be equal, which is not optimum with respect to the code parameters, we still see a clear advantage for sorting when the two bit error rates are different.

We thus conclude that sorting should be applied for multicarrier modulation in combination with 'tone ordering' such that the bits from carriers with the same bit load are gathered in RS symbols.

Along these lines we also studied the bit-sorting for the different bits of Gray-coded QAM.

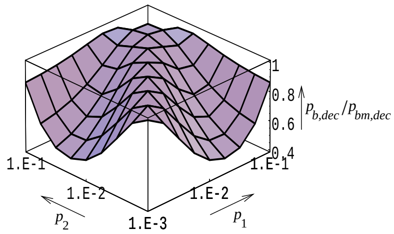

Figure 1: BERs after decoding of a (40,24) RS code over GF(2^6) as a function of the BERs before dec. p_i, $i = 1,2$

III. Bit-sorting for QAM?

Gray mapping is obtained recursively from a 4-QAM mapping by placing the previous smaller constellation into a quadrant and mirroring along the axis. Moving the points in the quadrants further apart (spacing $\alpha \geq 1$) leads to so-called hierarchical modulation. We obtain, *e.g.*, four different bit error rates p_i for the bits of a 256-QAM. Apart from the additional spacing, the differences are due to the number of points that have a nearest neighbor with a difference in the considered bit. These numbers are proportional to powers of 2 for Gray-coded mapping:

$$p_i = \frac{2 \cdot 2^{i-1}}{16} \cdot \begin{cases} p_\alpha\,, & i = 1 \\ p_0\,, & i \geq 2 \end{cases}\,, \tag{4}$$

$$p_\alpha = 1/2 \cdot \mathrm{erfc}(\alpha\sqrt{\mathrm{SNR}_0})\,,\ p_0 = 1/2 \cdot \mathrm{erfc}(\sqrt{\mathrm{SNR}_0})\,. \tag{5}$$

SNR_0 is the SNR with respect to the distance of nearest points in a QAM. Extending (3) to four different bit error rates, we are able to compute the corresponding bit error performance after decoding. In Fig. 2 we see that, unfortunately, we do not gain much (*cf.*, $P_{b,dec}$ and $P_{bm,dec}$) by sorting the bits according to their error probabilities. The differences in bit error rates have to be more pronounced than those provided by Gray coding, even if some bits are more protected by an additional spacing in a hierarchical scheme. Sorting bits into separate codewords rather than just symbols does yield noticeable performance differences as demonstrated by curves 1-4 in Fig. 2.

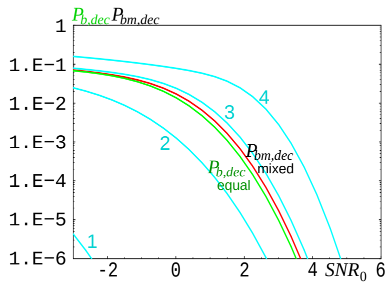

Figure 2: BERs after decoding of a (40,24) RS code over GF(2^6) of Gray-coded 256-QAM ($\alpha = 1.5$)

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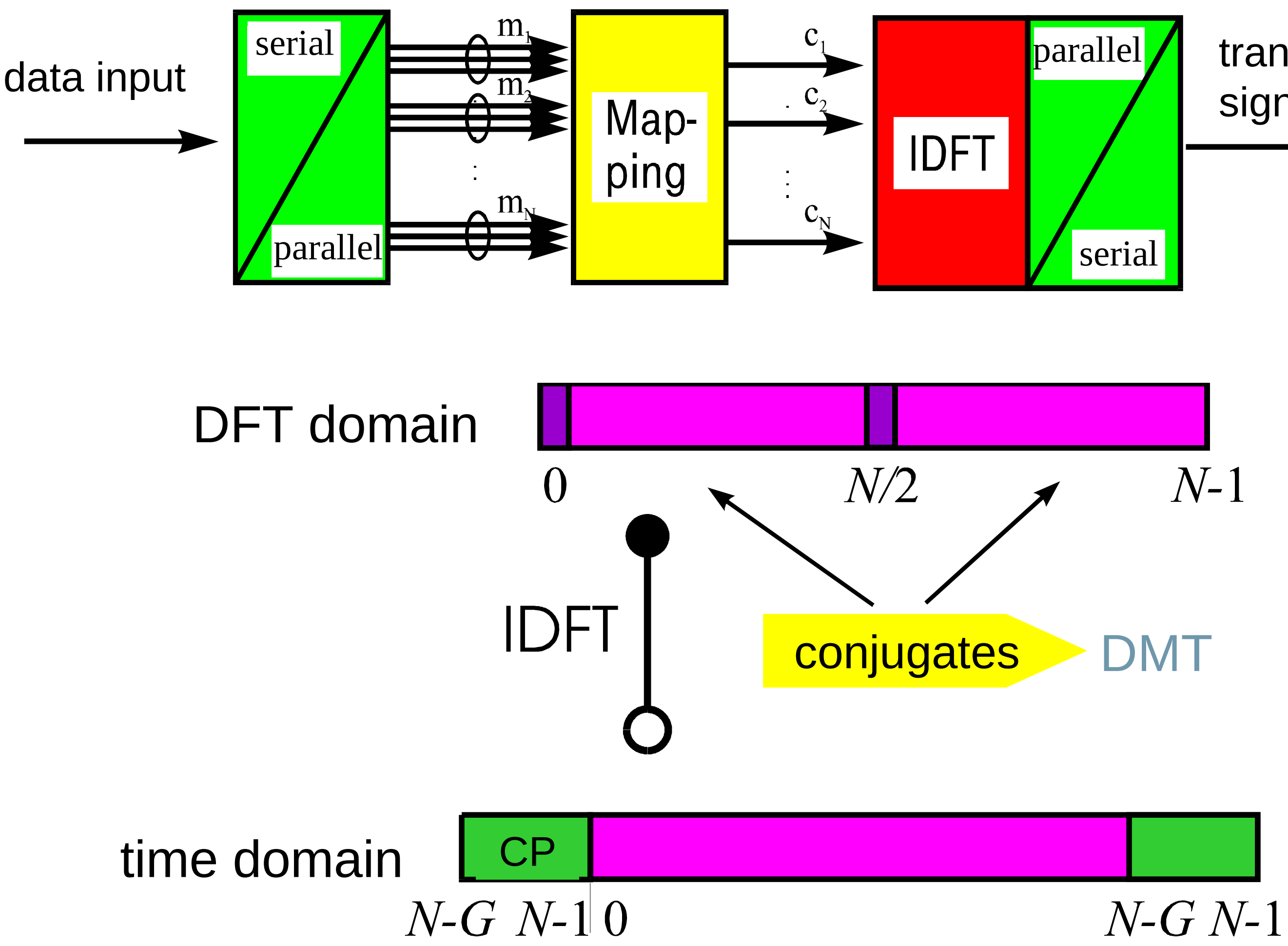
Sorting Bits into RS symbols …

- Introduction – what is tone ordering?
- Case with two different bit-error probabilities
- Sorting as a simple coded-modulation scheme?

Introduction – what is tone ordering?
OFDM and DMT



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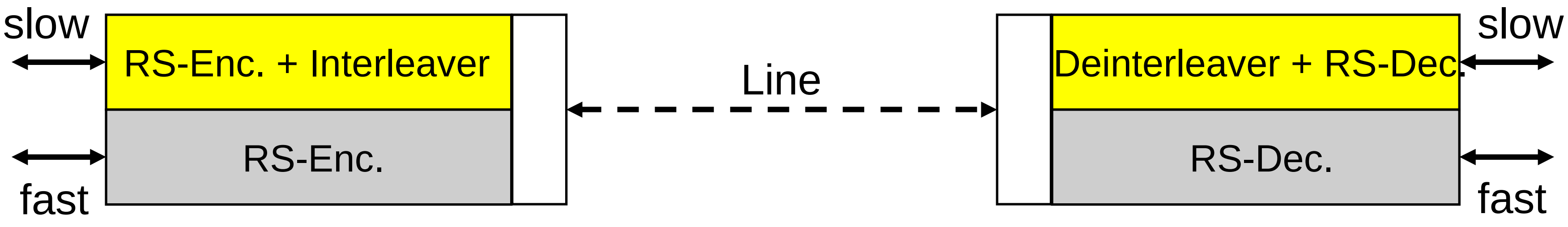
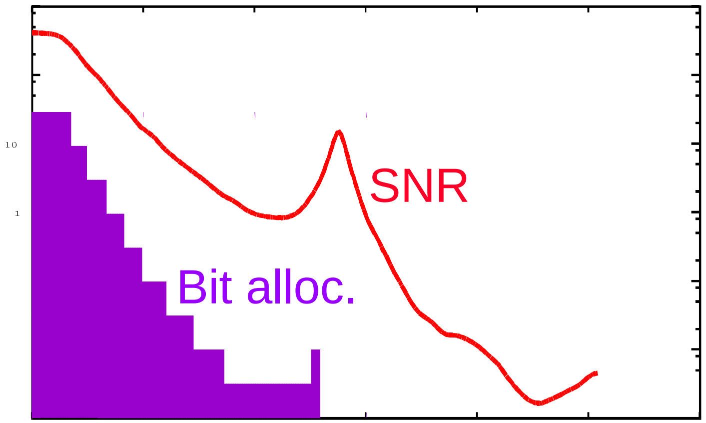
Introduction – what is tone ordering?

Tone ordering in ADSL





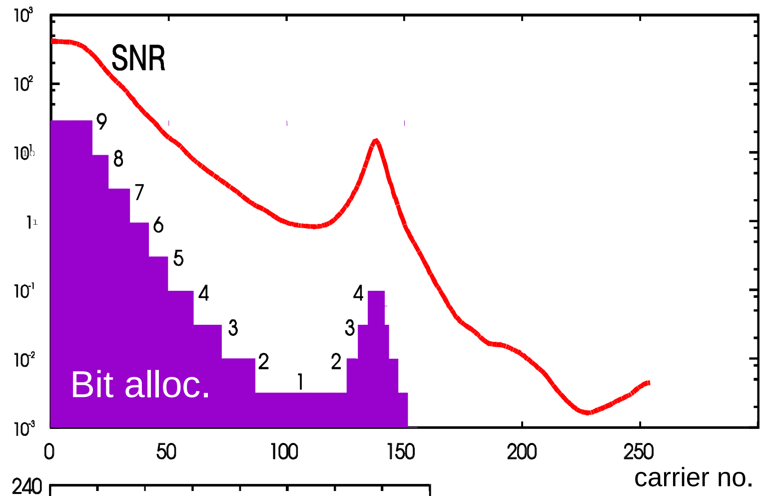
Route the low-SNR bits through the fast channel with less protection! !

Why not sorting it into the RS symbols according to their reliability? ?

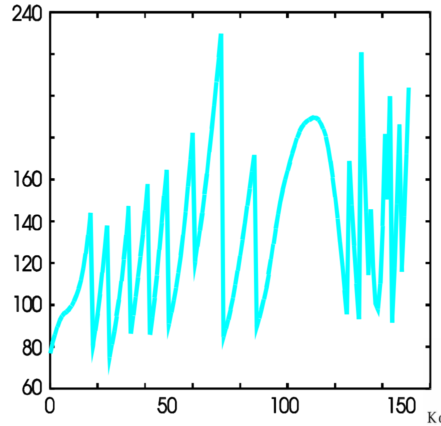
Introduction – what is tone ordering?
Tone ordering in ADSL

Non-stationary noise types to consider:

- switching on a disturbing alien system
- impulse noise



Non-equal bit error probabilities, if power allocation is not applied



power allocation

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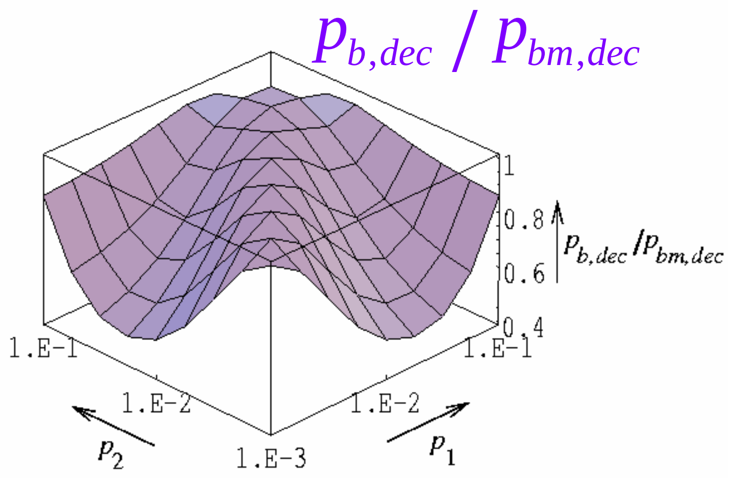
Two different bit error probabilities

Symbol error rates from bit error rates:

$$p_{si} = 1 - (1 - p_i)^m \qquad p_{sm} = 1 - \prod_{i=1}^{2} (1 - p_i)^{m/2}$$

Bit error rates after RS decoding:



$$p_{bm,dec} \approx \sum_{j=t+1}^{N} \frac{j}{N} \cdot \frac{p_{bm}}{p_{sm}} \cdot \binom{N}{j} p_{sm}{}^{j} (1 - p_{sm}{}^{N-j})$$

$$p_{b,dec} \approx \sum_{j=t+1}^{N} \sum_{j_1=0}^{\min(j,N/2)} \binom{N/2}{j_1} p_{s1}{}^{j_1} (1 - p_{s1}{}^{N/2-j_1}) \cdot$$

$$\cdot \binom{N/2}{j - j_1} p_{s2}{}^{j-j_1} (1 - p_{s2}{}^{N/2-j+j_1}) \left(j_1 \frac{p_{b1}}{p_{s1}} + (j - j_1) \frac{p_{b2}}{p_{s2}} \right) / N$$

Sorting as coded modulation?

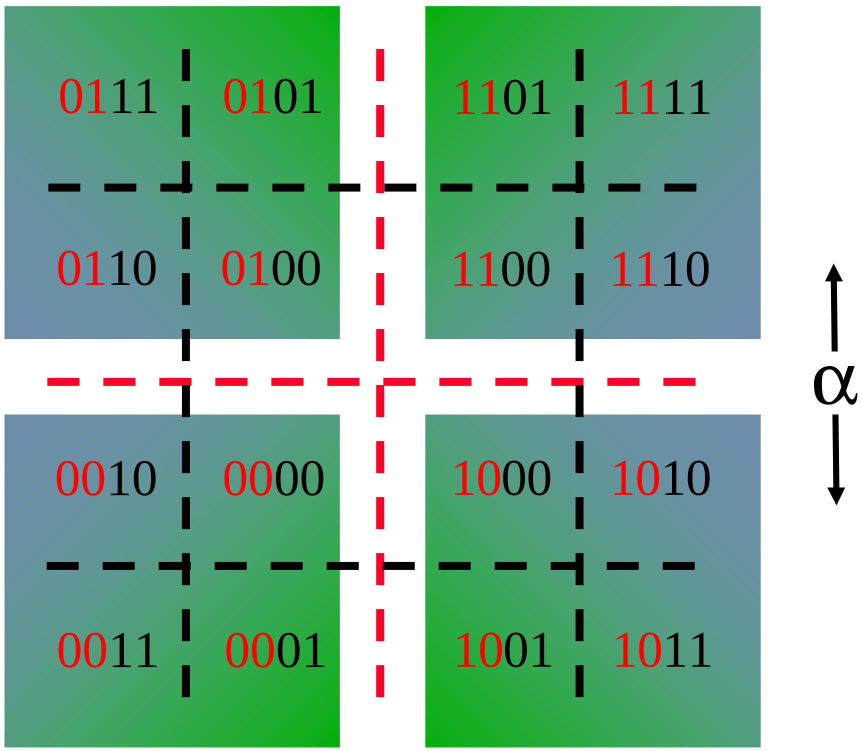
Gray code by mirroring along the axes:

0111	0101	1101	1111
0110	0100	1100	1110
0010	0000	1000	1010
0011	0001	1001	1011

Twice as much transitions for the bits in black!

Sorting as coded modulation?

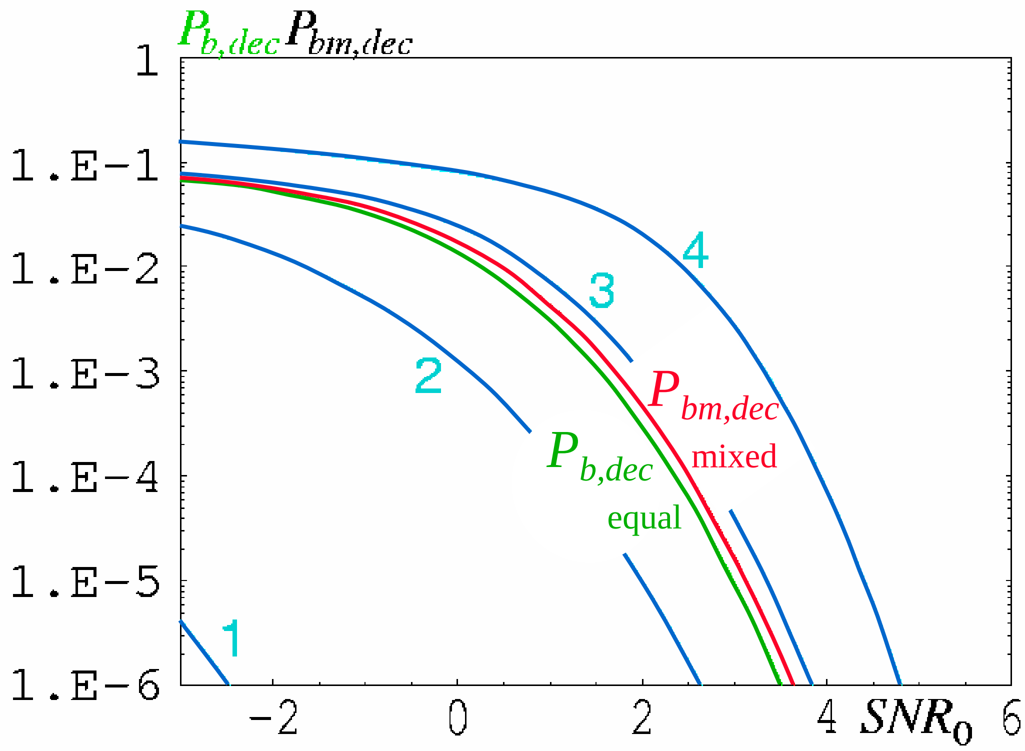
Stronger difference in a hierarchical QAM:



Sorting as coded modulation?

Example: 256-QAM

$$p_i = \frac{2 \cdot 2^{i-1}}{16} \cdot \begin{cases} 1/2 \cdot \mathrm{erfc}\left(\alpha\sqrt{\mathrm{SNR}_0}\right), & i = 1 \\ 1/2 \cdot \mathrm{erfc}\left(\sqrt{\mathrm{SNR}_0}\right), & i > 1 \end{cases}$$



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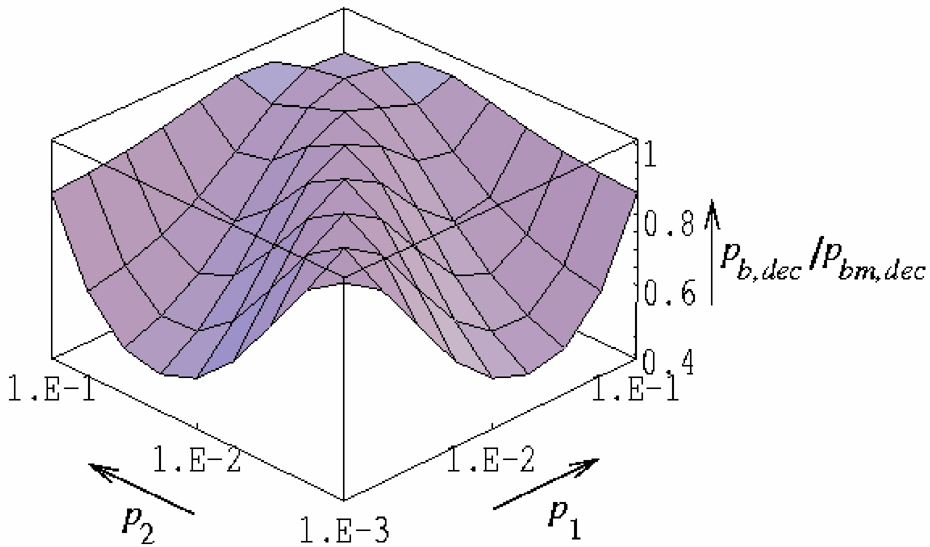
Conclusions

- Tone ordering should also gather bits from carriers with similar SNR into the same RS symbols



- Unfortunately, the bit-error rate differences in a QAM pattern do not suffice to yield a substantial gain when bits are sorted into RS symbols according to their reliability.

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