A Linear Piecewise Suboptimum Detector for Signals in Class-A Noise

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***Abstract*—In this paper, we consider the detection problem of binary signals corrupted by Class-A interference for two observations per symbol. The Class-A density contains infinitely many terms of scaled Gaussian-mixture densities, which yields an optimum detector that requires a high computational complexity. The linear (Gaussian) detector can be used, but it suffers from a significant performance degradation in stong impulse environments. The main objective of this paper is to design a simple detector with optimum performance. We start from the optimum decision boundaries, where we propose a piecewise linear approximation for nonlinear regions. As a result, we introduce a novel piecewise detector, which has much less complexity compared with the optimum one. Simulation results show a near-optimal performance for the proposed detectors in different impulse channel environments. Moreover, we show that one and two piecewise linear approximation per each nonlinear region is sufficient to approach the optimum performance.**

***Index Terms*—Impulse interference, Class-A density, piecewise linear approximation.**

I. Introduction

Non-Gaussian interference is frequently considered in many wireless communication systems due to the different interference emissions from the surrounding sources, such as atmospheric noise, man-made electromagnetic interference, and ignition [1], [2]. Moreover, it has been seen that laptop and desktop computers, computing platform subsystems clocks and busses generate significant non-Gaussian radio frequency interference (RFI) for the embedded wireless data transceivers (WLAN, IEEE 802.11b/g) [3].

The additive white Gaussian noise (AWGN) model is quite inadequate when the dominant source of interference contains impulse noise components. There are several distributions to model non-Gaussian interference, such as Middleton's models [2], the Symmetric Alpha-Stable distribution [4], [5], and the Gaussian mixture distribution [6]. Middleton's Class-A model represents one of the most applied models for narrowband impulse interference in communication systems. Since the parameters of this model are directly related to the underlying physical mechanism, it is widely used for statistical-physical modeling of RFI and co-channel interference [2], [7], [8].

The maximum likelihood (ML) detector for binary signals in Class-A interference has a non-reducible likelihood ratio test (LRT), which requires complex computations. As a suboptimum solution, a linear detector can be used, which is optimum for Gaussian noise. However, it has a very poor performance compared with the optimum detector in a strong impulse noise. In [1], a locally optimum detector (LOD) is introduced to reduce the receiver complexity and provide a near-optimum performance for a small signal assumption. However, its performance degrades dramatically at a high signal-to-noise ratio (SNR). In [9], we showed a wide area of nonlinear boundaries for Class-A interference, which results from the heavy-tailed property of the impulse noise distribution. In this paper, the design of the proposed detector depends on the decision boundaries of the approximated ML detector. Based on the closed-form expressions of the decision boundaries, we propose an efficient piecewise linear approximation of the nonlinear regions. Moreover, we provide an analytical evaluation of the error probability for the proposed detectors.

This paper is organized as follows. Section II briefly describes the system model and background. In Section III, we introduce the piecewise linear detectors. In Section IV, the error probability of the proposed detectors is evaluated. Finally, simulation results and concluding remarks are presented in sections V and VI, respectively.

II. System Model And Background

A. System Model

We consider a classical detection problem of binary signals corrupted by non-Gaussian interference. For simplicity, we restrict our analysis to binary PSK. However, the generalization to an arbitrary M-ary signaling is straightforward. Moreover, we assume the receiver has a priori knowledge of the exact impulse noise parameters. This is a reasonable assumption, since it has been shown that reliable estimates can be extracted from noisy samples [7] and can be applied in real communication systems. We further assume that the observation space consisting of two samples per symbol with independent noise samples. The received two samples may result from either employing a 2-path diversity or oversampling by a factor of 2. Hence, the received signal vector is $\mathbf{r} = [r_1 r_2]$, where

$$r_k = s + z_k\,, \quad k = 1, 2\,. \tag{1}$$

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$s \in \pm B$ is a transmitted antipodal symbol, and z_k, $k = 1, 2$ are i.i.d. Class-A noise samples. A Class-A density is a scaled mixture of Gaussian distributions and can be expressed as [1]

$$f_z(z) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} g(z; 0, \sigma_m^2) \ , \tag{2}$$

where

$$g(z; \mu, \sigma_m^2) = \frac{1}{\sqrt{2\pi\sigma_m^2}} e^{-\frac{(z-\mu)^2}{2\sigma_m^2}} \tag{3}$$

and

$$\sigma_m^2 = \frac{m/A + \Gamma}{1 + \Gamma} \ . \tag{4}$$

The Class-A density is well defined by the two parameters $A \in [10^{-2}, 1]$, and $\Gamma \in [10^{-6}, 1]$. The impulsive index, A, and the Gaussian factor, Γ, describe the intensity with which impulse events occur and the power ratio of the Gaussian and non-Gaussian components, respectively. In [1], the optimum detector for binary signals corrupted by Class-A interference is evaluated for an arbitrary sample size of N samples per symbol. For equiprobable transmitted symbols, the optimum ML detector computes the following LRT:

$$\begin{aligned} \Lambda(r) &= \frac{f_z(r_1 - B) f_z(r_2 - B)}{f_z(r_2 + B) f_z(r_2 + B)} \underset{H_0}{\overset{H_1}{\gtrless}} 1, \\ &= \frac{\prod_{k=1}^{2} \sum_{m=0}^{\infty} \frac{A^m}{m!} g(r_k; B, \sigma_m^2)}{\prod_{k=1}^{2} \sum_{m=0}^{\infty} \frac{A^m}{m!} g(r_k; -B, \sigma_m^2)} \underset{H_0}{\overset{H_1}{\gtrless}} 1 \ , \end{aligned} \tag{5}$$

where the hypotheses H_1 and H_0 correspond to $s = +B$ and $s = -B$, respectively. It is clear from (5) that the optimum detector requires a high computational complexity. In practice, the infinite sum in the Class-A density may be truncated into a finite sum. It was shown in [6] that the Class-A density can be well approximated by a two-term model

$$f_z(z) = e^{-A} g(z; 0, \sigma_0^2) + (1 - e^{-A}) g(z; 0, \sigma_1^2) \ . \tag{6}$$

The optimum detector based on the two-term approximation of a Class-A density is still requiring a high complexity. Since the optimum detector computes two exponential functions for all possible hypotheses. To reduce the detector complexity, we introduced in [9] an efficient approximation for the two-term Class-A density as follows:

$$f_z(z) \approx \begin{cases} e^{-A} g(z; 0, \sigma_0^2) & \text{if } \ , -k_0 \leq z \leq k_0 \ , \\ (1 - e^{-A}) g(z; 0, \sigma_1^2) \ , & \text{otherwise} \ , \end{cases} \ , \tag{7}$$

where $k_0 = \sqrt{\frac{2\sigma_0^2\sigma_1^2}{\sigma_1^2 - \sigma_0^2} \ln\left(\frac{\sigma_1 e^{-A}}{\sigma_0 (1 - e^{-A})}\right)}$ denotes the threshold at which the two terms are equal. For different impulse noise environments, we showed in [9] that the performance of a ML detector that uses (7) for computing its LRT has almost the same performance of the optimum one that uses a full Class-A density.

B. Optimal decision regions

In [9], we derived the decision boundaries of the ML detector that uses the approximate density (7). Moreover, these decision boundaries are used as a new approach to explain the behavior of many suboptimum detectors such as a linear detector, a LOD, and a soft limiter. Figure 1 shows the decison regions for $A = 0.3$ and $\Gamma = 0.08$. Since the decison boundaries in the second and fourth quadrant are identical, we only present the analytical results for the regions in the second quadrant. In region R_1, the decision boundary is expressed as [9]

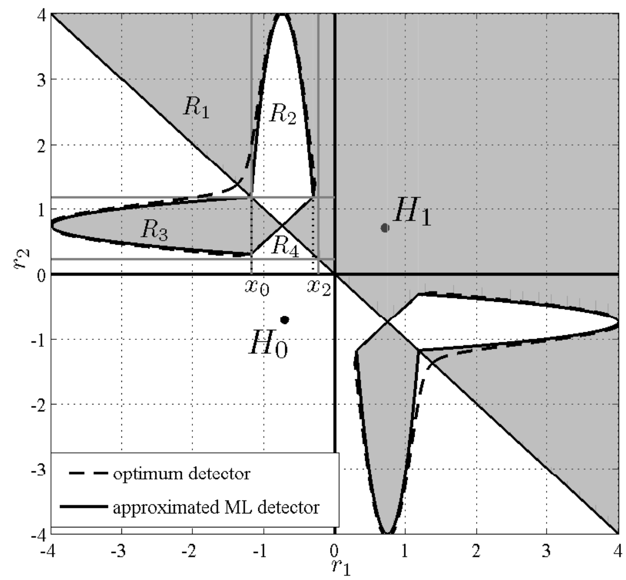

Fig. 1. Decision regions with $A = 0.3$, $\Gamma = 0.08$ at an SNR = 0 dB for $B = 1/\sqrt{2}$. Shaded area: decide for H_1, white area: decide for H_0.

$$r_2 = -r_1 \ . \tag{8}$$

In region R_2, the decision boundary has the following solution

$$r_2 = \frac{\sigma_1^2 - \sigma_0^2}{4B\sigma_0^2} \{k_0^2 - (r_1^2 + 2B\frac{\sigma_1^2 + \sigma_0^2}{\sigma_1^2 - \sigma_0^2} r_1 + B^2)\} \ . \tag{9}$$

In region R_3, the decision boundary equation is as follows

$$r_1 = \frac{\sigma_0^2 - \sigma_1^2}{4B\sigma_0^2} \{k_0^2 - (r_2^2 - 2B\frac{\sigma_1^2 + \sigma_0^2}{\sigma_1^2 - \sigma_0^2} r_2 + B^2)\} \ . \tag{10}$$

In region R_4, the decision boundary is

$$r_2 = -r_1 \ , \tag{11}$$

$$r_2 = r_1 + 2B\frac{\sigma_1^2 + \sigma_0^2}{\sigma_1^2 - \sigma_0^2} \ . \tag{12}$$

As we see in Fig. 1 the approximate ML detector yields similar decision boundaries to the optimum one (evaluated numerically). Since the approximated ML detector offers a closed-form expression for the decision regions, we consider it for introducing a piecewise linear detector.

III. Piecewise Linear Suboptimal Detectors

The proposed suboptimal detectors are based on replacing a nonlinear decision boundary with a piecewise linear function. This approximation allows the detector to compute a linear metric, which only requires computations similar to those of a linear detector. As we can see from (9) and (10), the nonlinear boundaries of the second quadrant involve quadratic equations. Since the quadratic equations are similar, we present the analytical piecewise linearization of the decision boundary defined in (9). The quadratic equation of (9) can be written as

$$f(x) = x^2 + 2B\frac{\sigma_1^2+\sigma_0^2}{\sigma_1^2-\sigma_0^2}x + B^2, \; x_0 \le x \le x_2 \;, \tag{13}$$

where x_0 can be obtained by solving (11) and (9) to yield the solution

$$x_0 = -B - k_0 \;, \tag{14}$$

and x_2 can be calculated as the intersection point of (12) and (9) to be as

$$x_2 = -B + k_0 - \frac{4B\sigma_0^2}{\sigma_1^2-\sigma_0^2} \;, \tag{15}$$

which define the interval of $f(x)$ as shown in Fig. 2. In the interval $x_{i-1} \le x \le x_i$, our objective is to determine the linear segment, $f_{s_i}(x) = m_i x + c_i$, that approximates $f(x)$. A linear approximation means the choice of the slope m_i, and the offset c_i. We choose m_i and c_i such that the mean squared error, ε, is minimized. The mean squared error in the interval $x_{i-1} \le x \le x_i$ can be expressed as follows:

$$\varepsilon = \int_{x_{i-1}}^{x_i} |f(x) - f_{s_i}(x)|^2 dx \;. \tag{16}$$

This function can be easily minimized with respect to the coefficients m_i and c_i to yield the solutions

$$m_i = x_{i-1} + x_i + 2B\frac{\sigma_1^2+\sigma_0^2}{\sigma_1^2-\sigma_0^2} \;, \tag{17}$$

and

$$c_i = B^2 - \frac{x_{i-1}^2 + x_i^2 + 4x_{i-1}x_i}{6} \;. \tag{18}$$

When more than one segment is used to approximate $f(x)$, the segment spacing Δ_i can be computed by solving $f_{s_i}(x_i) = f_{s_{i-1}}(x_i)$ to yield the following solution

$$x_i = \frac{x_{i-1} + x_{i+1}}{2} \;, \tag{19}$$

and hence, we have

$$\Delta_i = \Delta_{i+1} = \frac{x_{i+1} - x_{i-1}}{2} \;, \tag{20}$$

which results in equally spaced subintervals. Notice that when $f(x)$ is not a quadratic function then it is not necessarily that the solution of (19) yields equally spaced subintervals.

A. One-piece Linear approximation

This detector uses a one linear segment to approximate each nonlinear decision region. The one-piece approximation of $f(x)$ is given as

$$f_1(x) = B^2 - \frac{x_0^2 + x_2^2 + 4x_0x_2}{6} \;, \tag{21}$$

which assumes the nonlinear boundary of the interval $x_0 \le x \le x_2$ as a linear one as shown in Fig. 2. Hence, the decision boundary of the region R_2 can be approximated as

$$\begin{cases} s = H_1 & \text{if } r_2 > \frac{\sigma_1^2-\sigma_0^2}{4B\sigma_0^2}\{k_0^2 - f_1(x)\} \\ s = H_0 & \text{if } r_2 \le \frac{\sigma_1^2-\sigma_0^2}{4B\sigma_0^2}\{k_0^2 - f_1(x)\} \end{cases} \;, \tag{22}$$

B. Two-piece Linear Approximation

This detector uses two linear segments to approximate the function $f(x)$ over the interval $x_0 \le x \le x_2$. As we see in Fig. 2, the first segment, $f_{s_1}(x)$, and the second segment, $f_{s_2}(x)$, approximate $f(x)$ over subintervals $x_0 \le x \le x_1$, and $x_1 \le x \le x_2$, respectively. x_1 can be calculated analytically using (19), which corresponds to the intersection point of (11) and (12), and can be expressed as

$$x_1 = -B\frac{\sigma_1^2+\sigma_0^2}{\sigma_1^2-\sigma_0^2} \;. \tag{23}$$

the two-piece function $f_2(x)$ is then given as

$$f_2(x) = \begin{cases} m_1x + c_1 & \text{if } x_0 \le x \le x_1 \\ m_2x + c_2 & \text{if } x_1 \le x \le x_2 \end{cases} \;, \tag{24}$$

where the coefficients (m_1, c_1), and (m_2, c_2) can be obtained analytically by substituting (14), (15), and (23) into (17) and (18). The previous analysis can be easily extended to an arbitrary number of segments. However, as we will show in the simulation results, the two-piecewise approximation is already sufficient for a near optimum performance.

IV. Performance Evaluation

Our objective in this section is to compute the error probability, P_e, for the considered detectors using a decision boundary plot. According to (7), the probability densities on the hypothesis H_0, $p(\mathbf{r}|H_0)$, are labeled in Fig. 3. Now, P_e is simply the integral over $p(\mathbf{r}|H_0)$ in the hypothesis H_1 regions. Assuming equiprobable transmitted symbols, the expression for p_e is

$$\begin{aligned} p_e = & \iint_{R_{e_0}} (1-e^{-A})^2 g(r_1; -B, \sigma_1^2) g(r_2; -B, \sigma_1^2) d\mathbf{r} \\ & + 2\iint_{R_{e_1}} (1-e^{-A})^2 g(r_1; -B, \sigma_1^2) g(r_2; -B, \sigma_1^2) d\mathbf{r} \\ & + 2\iint_{R_{e_2}} e^{-A}(1-e^{-A}) g(r_1; -B, \sigma_0^2) g(r_2; -B, \sigma_1^2) d\mathbf{r} \\ & + 2\iint_{R_{e_3}} (1-e^{-A})^2 g(r_1; -B, \sigma_1^2) g(r_2; -B, \sigma_1^2) d\mathbf{r} \,. \end{aligned} \tag{25}$$

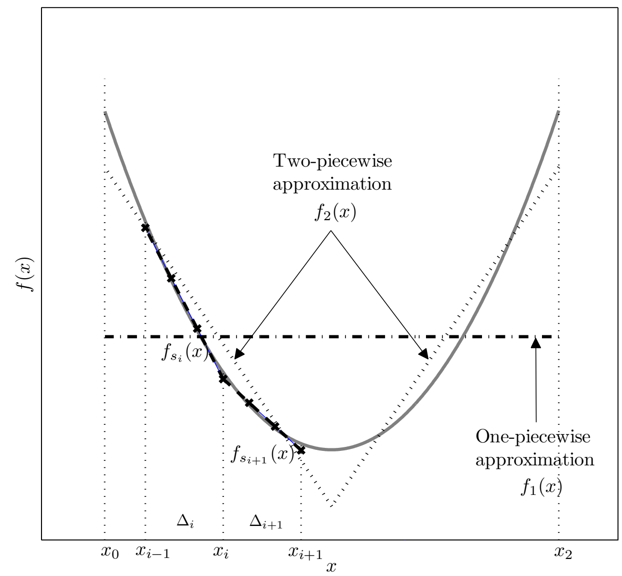

Fig. 2. Piecewise linear approximation of the function $f(x)$.

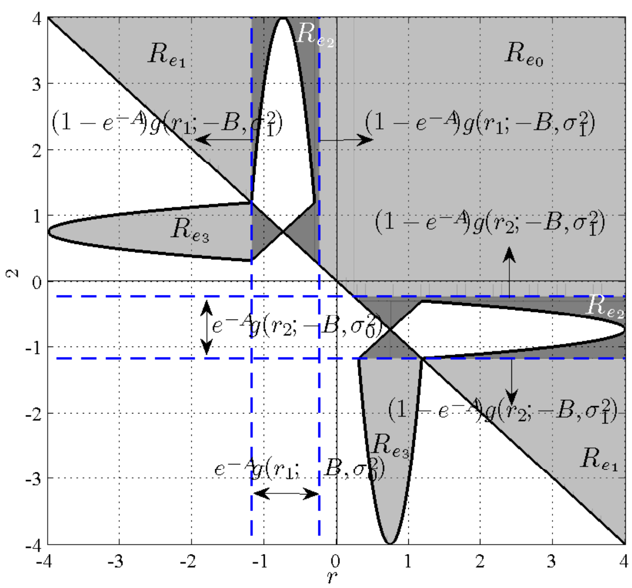

Fig. 3. Decision space of probability densities for the hypothesis H_0.

Clearly, we evaluate each integral over its regions. Labeling these integrals I_0, I_1, I_2, and I_3, after some mathematical steps, we obtain the following evaluation

$$
\begin{aligned}
I_0 =&(1-e^{-A})^2\{\frac{1}{4}\mathrm{erfc}^2(\frac{B}{\sqrt{2\sigma_1^2}}) \\
&+\frac{1}{\sqrt{2\pi\sigma_1^2}}\int_{r_1=-B+k_0}^{0} e^{-\frac{(r_1+B)^2}{2\sigma_1^2}}\mathrm{erfc}(\frac{B-r_1}{\sqrt{2\sigma_1^2}})dr_1\}\,,
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
I_1 =&(1-e^{-A})^2\{\frac{1}{2}\mathrm{erfc}(\frac{2B+k_0}{\sqrt{2\sigma_1^2}})\mathrm{erfc}(\frac{k_0}{\sqrt{2\sigma_1^2}}) \\
&-\frac{1}{\sqrt{2\pi\sigma_1^2}}\int_{r_1=B+k_0}^{\infty} e^{-\frac{(r_1+B)^2}{2\sigma_1^2}}\mathrm{erfc}(\frac{r_1-B}{\sqrt{2\sigma_1^2}})dr_1\}\,,
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
I_2 =&(1-e^{-A})\{\frac{e^{-A}}{\sqrt{2\pi\sigma_0^2}}\int_{r_1=x_2}^{-B+k_0} e^{-\frac{(r_1+B)^2}{2\sigma_0^2}}\mathrm{erfc}(\frac{B-r_1}{\sqrt{2\sigma_1^2}})dr_1 \\
&+\frac{2e^{-A}}{\pi\sqrt{\sigma_0^2\sigma_1^2}}\int_{r_1=x_0}^{x_1}\int_{r_2=r_1-2x_1}^{-r_1} e^{\frac{-(r_1+B)^2}{2\sigma_0^2}-\frac{(r_2+B)^2}{2\sigma_1^2}}dr_1dr_2 \\
&+\frac{2e^{-A}}{\pi\sqrt{\sigma_0^2\sigma_1^2}}\int_{r_1=x_0}^{x_1}\int_{r_2=\phi(r_1)}^{\infty} e^{\frac{-(r_1+B)^2}{2\sigma_0^2}-\frac{(r_2+B)^2}{2\sigma_1^2}}dr_1dr_2\}\,,
\end{aligned}
\tag{28}
$$

and

$$
I_3 = 2\frac{(1-e^{-A})^2}{\pi\sigma_1^2}\int_{r_1=x_0}^{x_1}\int_{r_2=-x_0}^{\phi(r_1)} e^{\frac{-(r_1+B)^2-(r_2+B)^2}{2\sigma_1^2}}dr_1dr_2\},
\tag{29}
$$

where erfc is the error function complement, and $\phi(r_1)$ is the nonlinear function of the region R_2 over the interval $x_0 \leq r_1 \leq x_1$. For the approximated ML detector, $\phi(r_1)$ is given as in (9). In the case of the proposed piecewise detectors, $\phi(r_1)$ can be expressed as

$$
\phi(r_1) = \frac{\sigma_1^2-\sigma_0^2}{4B\sigma_0^2}\{k_0^2 - f_i(r_1)\}\,, \quad i=1,2\,, \tag{30}
$$

where $f_1(x)$ and $f_2(x)$ are given in (21) and (24), respectively.

V. Simulation Results

In this section, we evaluate the bit-error ratio (BER) of binary signals in the presence of Class-A interference. The error probability of the optimum and proposed detectors is evaluated by simulation for different parameters of Class-A density. In the simulation, we use 100 terms to closely approximate the probability density function of Class-A interference.

Figure 4 provides simualtion results for a low-implusive case (with $A = 0.3$, and $\Gamma = 0.08$). It is clear from the shown figure that the optimum detector does not offer much better improvement over a linear detector at a high SNR. This performance is expected, since at high SNR the optimal decision boundaries become closer to those of the linear detector [9]. For this reason, a floor on the BER will not be expected for the one-segment linear piecewise detector when the SNR goes to infinity. The performance of the piecewise detectors (one-segment and two-segment) is also close to the optimal performance. Figures 5, and 6 depict the performance evaluation for moderate and strong impulse cases, respectively. It is clear from both figures that the performances of the proposed piecewise detectors approach the optimum detector.

Although the proposed piecewise detector is designed for the approximated Class-A density of (7), it shows a near optimum performance over Class-A interference with a full Class-A density (100 terms). Since our analysis in Section III can be extended to an arbitrary number of segments, it is clear from the simulation results that the piecewise detector with one-segment is sufficient to provide an almost optimum performance.

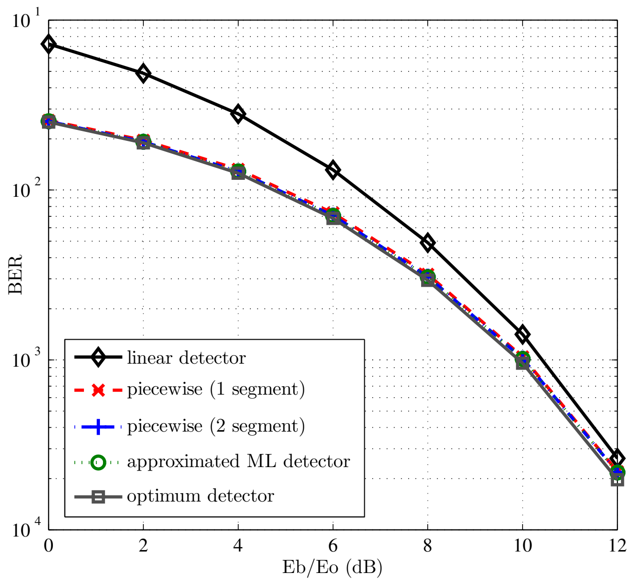

Fig. 4. Performance comparison over a low-impulsive channel with $A = 0.3$ and $\Gamma = 0.08$

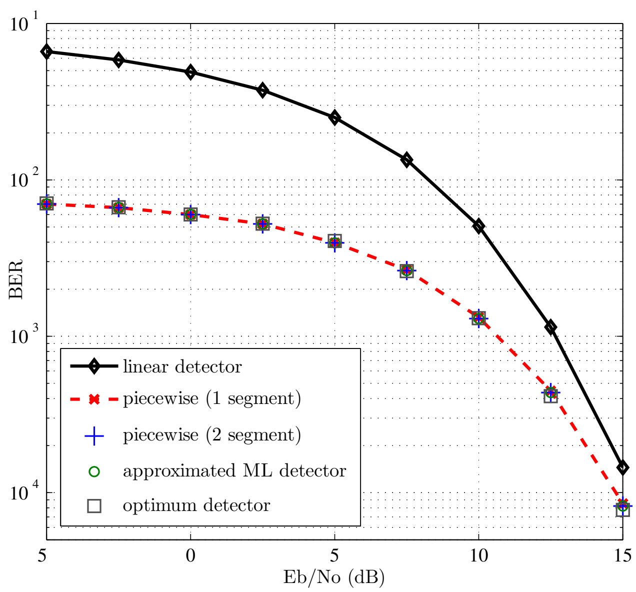

Fig. 5. Performance comparison over a moderately impulsive channel with $A = 0.1$ and $\Gamma = 0.01$

VI. Conclusion

In this paper, we introduced a new piecewise suboptimum detector for binary signals in the presence of Class-A interference for two observations per symbol ($N = 2$). The proposed detector approximates the optimal decision boundaries to yield a low complexity detector. The analytical bit-error ratio evaluation is confirmed by simulation and they show

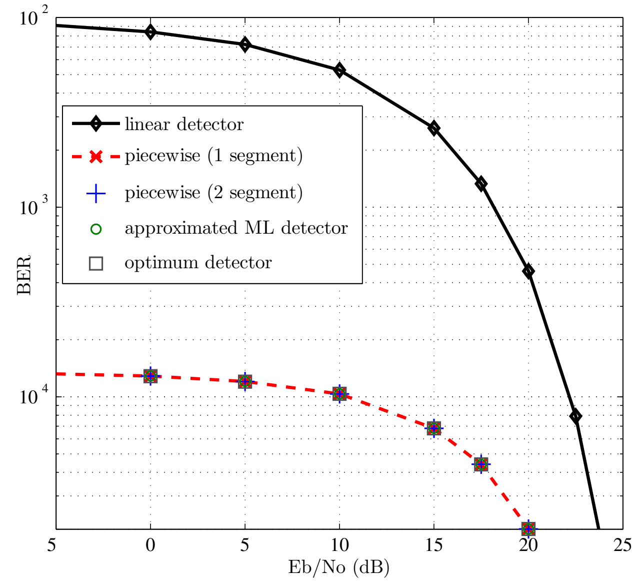

Fig. 6. Performance comparison over a highly impulsive channel with $A = 0.01$ and $\Gamma = 0.005$

that the piecewise linear detector provides an almost optimum performance for different impulse noise environments.

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