Analog Codes for Peak-to-Average Ratio Reduction

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Abstract — **Multi-tone modulation (DMT, Discrete Multitone, OFDM, Orthogonal Frequency Division Multiplex) has the disadvantage of a quite high peak-to-average ratio. Clipping of the high amplitudes caused by the analog circuitry leads to additional noise. In this contribution we show that in principal, so-called Analog Codes (Reed-Solomon Codes over complex numbers) can be used for eliminating this noise.**

I. Introduction

One of the drawbacks of multitone transmission is the quite high peak-to-average ratio (Crest factor). This is in accordance with the central limit theorem because in multitone signaling, a large number of carriers are combined leading to a near-Gaussian density. The high peak values are especially demanding for the analog circuitry. Either the peak values are clipped, leading to additional noise, out-of-band power, and non-linear echos or the analog front-ends must be designed with high dynamics, also requiring a high voltage power supply with the further disadvantage of high power consumption. Several countermeasures have been proposed so far:

1. block-wise rotation in the DFT domain and transmission of the time-domain sequence with the lowest peak-to-average ratio (denoted 'Partial Transmit Sequences') [1, 2, 3, 4, 5, 6],
2. modification of the DFT-domain vector with a set of pseudo-noise sequences and like before, transmission of the time-domain sequence with the lowest peak-to-average ratio (denoted 'Selected Mapping') [1, 4, 5, 7, 8, 9],
3. iterative representation of the peaks above the limit by redundant subspaces requiring an IFFT and FFT transform per iteration [10, 11, 12],
4. Trellis Shaping [13, 14, 15, 16], ,
5. use of Golay complementary sequences [17, 18, 19, 20, 21, 22],
6. use of Rudin-Shapiro sequences [20, 23],
7. direct use of m-sequences [24, 25, 26],
8. iterative improvement using smaller signal sets on some of the carriers [27],
9. conditioned pairs of carriers [28],
10. subtraction of Dirac-like functions in time domain ('tone reservation') [29, 30] ,
11. multiple representation of signal points in the DFT domain ('tone injection') [30, 31].

We will only comment on a few of these procedures. The first two proposals differ only in the block size for the addition of the pseudo-noise sequence. In the first proposal, the DFT frame is divided into blocks. Samples of the pseudo-noise sequence modify the whole block. The second proposal would thus have a block size of one. The use of trellis shaping (4) in DFT domain is only applicable for carrier numbers of up to 32 or 64 and small signal sets (4-QAM). Method 10 has the lowest complexity since no transformation is needed. It uses redundant DFT samples to define Dirac-like functions that are subtracted in time domain. The last method uses multiple QAM signal sets in DFT domain. One can thus choose from two or more equivalent signal points, thereby reducing the peak values in time domain.

In here, we first do not intend to avoid clipping by means of PAR reduction techniques but try to correct the clipping errors. Similar to some of the above-mentioned methods (e.g., procedure 10), we reserve some DFT samples for redundancy. If these are positioned consecutively (cyclically), we come to the structure of the so-called Analog Codes. These Codes and its application for clipping-noise correction are described in the following section. Thereafter, some first results are presented.

II. Analog Codes and its application to the clipping-noise correction

Reed-Solomon Codes can basically be defined as the values of a polynomial with limited degree $K-1$ at positions z^i, $i=0..N-1$, where z is an element of the order N. K is the number of information symbols and N is the codeword length. This definition is extended to such polynomials that have a number of $N-K$ zero coefficients that are cyclically following one another. We know from, e.g., the Lagrange interpolation formula that one only requires K of the N samples to define the original degree-$(K-1)$ polynomial. All this is true over every number field. The only requirement is that we need to have an element of the order N. Considering complex numbers, this element of order N, of course, is $z = e^{j\,2\pi/N}$, i.e., $z^N = \left(e^{j\frac{2\pi}{N}}\right)^N = 1 = z^0$. The definition of Analog Codes goes back to J.K. Wolf [32, 33, 34] and a detailed treatment can be found in [35].

In the extended definition, the samples of the analog

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codeword are given by

$$c_i = C(z^{-i}) = \sum_{k=0}^{K-1} C_k (z^{-i})^k (z^{-i})^m \qquad (1)$$

The clipping of DMT (OFDM) signals may be considered as adding impulse-like noise which disturbs only a few of the samples. As said before, from interpolation theory, we know that we are able to restore the RS polynomial by just picking $K-1$ arbitrary correct samples and applying a polynomial interpolation like the Lagrange approach. In coding theory, this procedure would be called erasure decoding, if the disturbed positions are known. We would propose not to compute the erroneous positions but simply detect them from its amplitude. I.e., we define a limit V_T at, e.g., 95 % of the clipping amplitude V_c and consider those samples to be unreliable that exceed this threshold. Keeping the threshold a little below the clipping amplitude allows for additional background noise.

In coding theory, the error value can be computed in time or DFT domain. Usually there, for systematic codes, the data is directly introduced in the time-domain sequence. Therefore, one would also prefer to correct the errors in time domain. In case of the DMT (OFDM) signaling, the information is located in the frequency domain. Thus, one may prefer a correction directly in DFT domain. This can be done by a recursive convolution derived from the so-called key equation. We will give a short sketch of the derivation.

First we define the error-locator polynomial $\Gamma(x)$ as

$$\Gamma(x) \;=\; \prod_{i \in \mathbb{F}} (x \;-\; z^{-i})\,, \qquad (2)$$

where $\mathbb{F}$ is the set of indices of the error positions ($\mathbb{F} = \{i | f_i \neq 0\}$, f_i being the components of the error vector). Correspondingly, let the non-error-locator polynomial be

$$\overline{\Gamma}(x) \;=\; \prod_{i \in \overline{\mathbb{F}}} (x \;-\; z^{-i})\,, \qquad (3)$$

where $\overline{\mathbb{F}}$ is the set of indices of the error-free positions ($\overline{\mathbb{F}} = \{i | f_i = 0\}$).

From $x^N - 1 = \prod_{i=0}^{N-1}(x - z^{-i})$ it follows that $\overline{\Gamma}(x) = (x^N - 1)/\Gamma(x)$. We obtain the key equation

$$\begin{aligned} \Gamma(x) \cdot F(x) \;&=\; \Gamma(x) \cdot T(x) \cdot \overline{\Gamma}(x) \;=\; T(x) \cdot (x^N - 1) \\ &=\; 0 \quad \text{mod } (x^N - 1)\,, \end{aligned} \qquad (4)$$

where $F(x)$ is the polynomial with DFT-domain coefficients corresponding to the error vector.

The product of the polynomials $\Gamma(x)$ and $F(x)$ corresponds to a cyclic convolution of its coefficients

$$\sum_{i=0}^{N-1} \Gamma_i \cdot F_{j-i} \;=\; \sum_{i=0}^{e} \Gamma_i \cdot F_{j-i} \;=\; 0\,, \quad j = 0, \ldots, N-1\,, \qquad (5)$$

e being the number of errors.

Noting that $\Gamma(x)$ is a monic polynomial ($\Gamma_0 = 1$), Eq. (5) yields a recursive extension of the syndrome, i.e., of the DFT components that are only dependent on the error.

$$S_j = -\sum_{i=1}^{e} \Gamma_i \cdot S_{j-i}\,, \quad j = N-K, \ldots N-1 \text{ mod } N \quad (6)$$

Where S_j equals F_j for the DFT coefficients that have originally been zero. After extension, the syndrome is subtracted from the DFT components.

III. Results

This sections gathers some first results for clipping correction with and without channel filter responses. Clipping is considered to take place either before or after the filter response. Two exemplary filter responses, a rectangular one and one taken from the Issue 2 ADSL standards document, have been selected.

In Fig. 1 we show a result for the clipping-correction gain in the case of no additional background noise for the dependency on the ratio of the clipping value V_c over the RMS value V_{RMS}. The clipping-correction gain G is defined to be

$$G = 10 \log_{10}(N_c / N_{corr})\,, \qquad (7)$$

which is a measure for the clipping-corrected noise N_{corr} relative to the clipping noise without correction N_c. The parameters used for all examples throughout this paper are: $N = 64$, 12 parity positions, 16-QAM on all used carriers. One should, however, not conclude that low rates should be essential. N was chosen as a quite small number just to shorten simulation times.

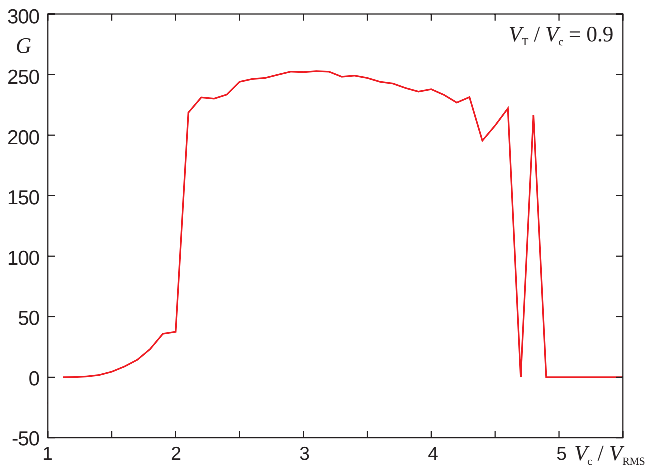

Fig. 1: Clipping-correction gain dependent on the ratio of the clipping amplitude relative to the RMS value without any filter response and without additional background noise

For small values of V_c/V_{RMS} the error-correcting capability of the code is exceeded and for quite high values clipping is not very likely. Thus, in these cases, the gain reduces to zero.

With additional background noise (AWGN) with an SNR of 50 dB, the gains reduce to more realistic values. Figure 2 shows the results for three different detection thresholds V_T. Figure 3 contains the corresponding average and maximum numbers of clipped samples.

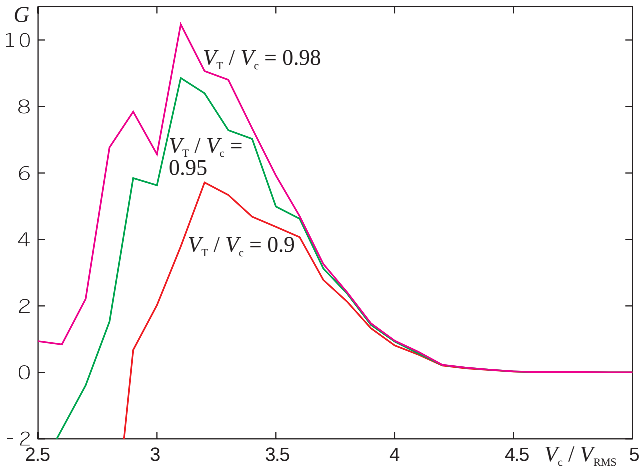

Fig. 2: Clipping-correction gain dependent on the ratio of the clipping amplitude relative to the RMS value without any filter response, but with added background noise

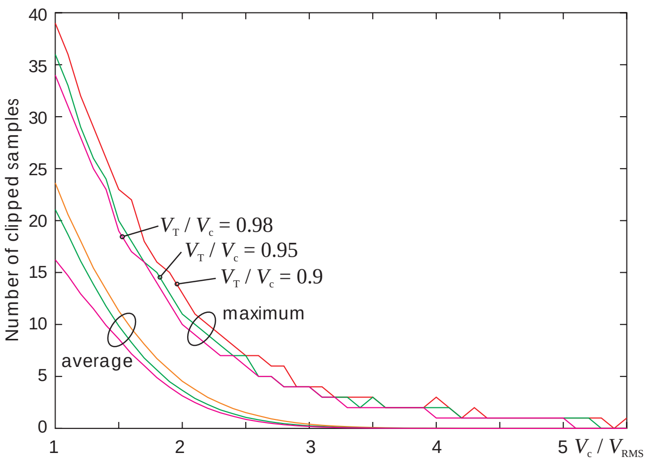

Fig. 3: Average and maximum numbers of clipped samples dependent on the ratio of the clipping amplitude relative to the RMS value

If the clipping occurs directly after the IFFT operation of the DMT modulation without any filtering in between, the results of Fig. 2 also hold for any kind of frequency response of the channel if an equalization is provided at the receiver before clipping correction. There will, however, be a regrowth of the PAR on the channel after filtering. The dependency on the clipping amplitude is shown in Fig. 4.

A filter response before the clipping operation, i.e., a

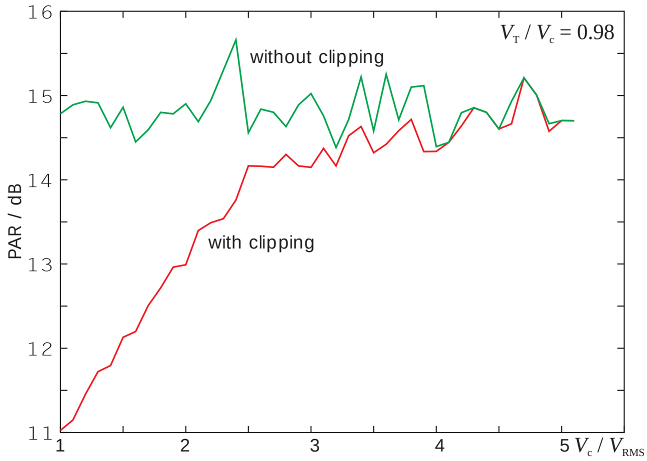

Fig. 4: PAR after filtering (rectangular) dependent on the ratio of the clipping amplitude relative to the RMS value

pulse shaping filter before the nonlinear amplifier will lead to a significantly higher PAR due to some overshooting. The clipping-reduction gains are much lower if we compare them to the gains in Fig. 2. Results with two different filter responses, a rectangular one and one following the ADSL standard, are gathered in Fig. 5. We see that the error correcting capability is easily exceeded if a filter response is applied before the clipping.

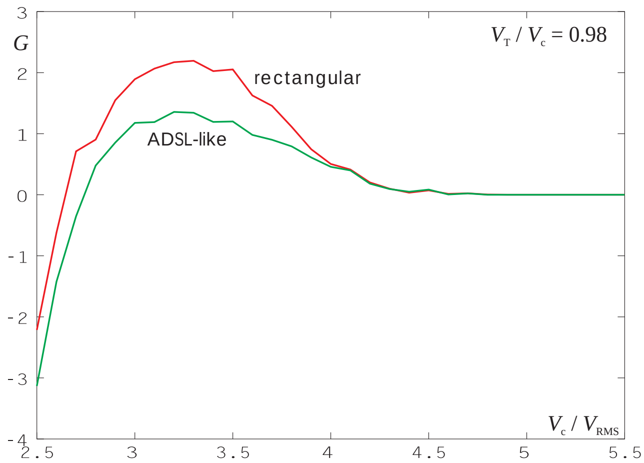

Fig. 5: Clipping-correction gain dependent on the ratio of the clipping amplitude relative to the RMS value with two different filter responses before clipping and additional background noise

IV. Conclusions

We have shown that Reed-Solomon Codes over complex numbers, so-called 'Analog Codes' are suited to correct clipping errors caused by nonlinear amplifiers. Such clipping errors which may be regarded as impulse noise is the type of noise analog codes are suited for. We studied cased with and without filter responses. Significant gains (several dB) can be obtained without filter responses or, equivalently, with a filter response after the clipping which is equalized at the input of the receiver. Any filter response prior to clipping leads to quite low gains (only 1 or 2 dB) which is due to overshooting caused by the filter response. This causes more and higher signal peaks and with it more clipping errors often exceeding the error correcting capability of the code. Thus, an important requirement for the application of Analog Codes is that all shaping filters should not precede the nonlinear operation. The application is thus excluded for wireless transmission (OFDM) where the nonlinearity is usually the last component of the transmitter signal path. Analog Codes for clipping correction should rather be suited for cable transmission (DMT) where some of the filters (splitters) are anyway positioned after the active amplifier circuits.

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