ABCD matrix model for a 3-wire power-line cable

Werner Henkel and Tai Him Yim

Constructor University Bremen
werner.henkel@ieee.org

Abstract

Typical in-house power-line cables have three wires for L, N, and PE. Hence, a two-port description requires four voltages and four currents, i.e., a 4×4 ABCD or S-parameter matrix. To obtain a simple description, one would like to extend the typical 2×2 ABCD matrix known for a 2-conductor transmission line.

Starting from a full S-parameter specification moved into ABCD, we show, how this matrix may be modeled starting from a 2×2 ABCD description based on secondary line parameters. This is obtained by the use of twisted pair FEXT modeling.

1 Matrix descriptions of a 3-wire transmission line

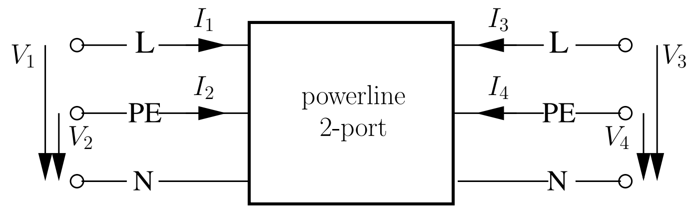

Figure 1: Power line two-port

A 3-wire two-port as shown in Fig. 1 has, of course, four voltages and 4 currents, likewise four incident and four reflected wave components.
We obtain the matrices

$$\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 & B_1 & B_2 \\ A_3 & A_4 & B_3 & B_4 \\ C_1 & C_2 & D_1 & D_2 \\ C_3 & C_4 & D_3 & D_4 \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \\ I_3 \\ I_4 \end{bmatrix} , \quad \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} . \tag{1}$$

Figure 2 shows all measurement combinations required to determine the full set of S-parameters, obviously leading to $4 \cdot 6 = 24$ parameters, only requiring 16. Hence, some are determined repeatedly and one may take the mean of such measurements.
Using the transform relations in [1], one can move from $\mathbf{S}$ to ABCD representation, possibly with $\mathbf{Z}$ as an intermediate step.
A usual 2-conductor transmission line is described by the well-known ABCD formulation

$$\begin{bmatrix} \cosh \gamma l & Z_w \sinh \gamma l \\ \frac{1}{Z_w} \sinh \gamma l & \cosh \gamma l \end{bmatrix} . \tag{2}$$

The secondary line parameters Z_w and $\gamma = \alpha + j\beta$ can be measured by open and short terminations, despite of some virtual length extension for open, thereby leading to small length deficiencies.

2 Model for the 4×4 ABCD matrix

In the first and fourth arrangement in Fig. 2, we recognize that they represent standard 2-wire pairs with some terminations at the other pairs added. The resulting parameters are easily described by (2), hence obtaining 8 parameters of the 4×4 ABCD matrix directly, making a checkerboard arrangement at positions 11, 13, 22, 24, 31, 33, 42, 44. The remaining parameters represent FEXT (Far-End Crosstalk) terms such as the one at position 12 refers to V_1/V_4 or position 41 refers to I_2/V_3. From DSL standards [2,3], we know a model for FEXT in the form of

$$H_{FEXT} = H_{TR} \cdot \sqrt{l/l_0} \cdot f \cdot k \quad (3)$$

with the transfer function H_{TR}, length l, frequency f, and a constant k. In the DSL case there is also a component due to the bigger arrangement with, e.g., 50 pairs. When adopting such dependencies, we can, e.g., formulate the two mentioned components using

$$\frac{V_1}{V_4} \approx \frac{V_2}{V_4}\sqrt{l/l_0} \cdot f \cdot k \text{ and } \frac{I_2}{V_3} \approx \frac{I_1}{V_3}\sqrt{l/l_0} \cdot f \cdot k \ . \quad (4)$$

The constant k can be determined comparing the component obtained from the full scattering matrix and the one through this model.
The intuitive reasoning for the FEXT model is that independent on the coupling location, the overall transfer function has to be in there, assuming identical transfer functions on both pairs. In power terms, the coupling should grow with the length, explaining $\sqrt{l/l_0}$. The frequency dependency would results from seeing the two loops as a “transformer” and there the mutual inductance comes with $j\omega$.
Figure 3 compares the results[1] obtained from scattering parameters (blue) with the ones (red) obtained directly from open/short measurements and the resulting 2×2 ABCD matrix or from the described modeling as shown in equations (4). This follows the described checkerboard pattern with almost exact matches and models that deviate a bit, due to the simplified modeling, which is especially deviating at lower frequencies. We will discuss this more during the workshop.

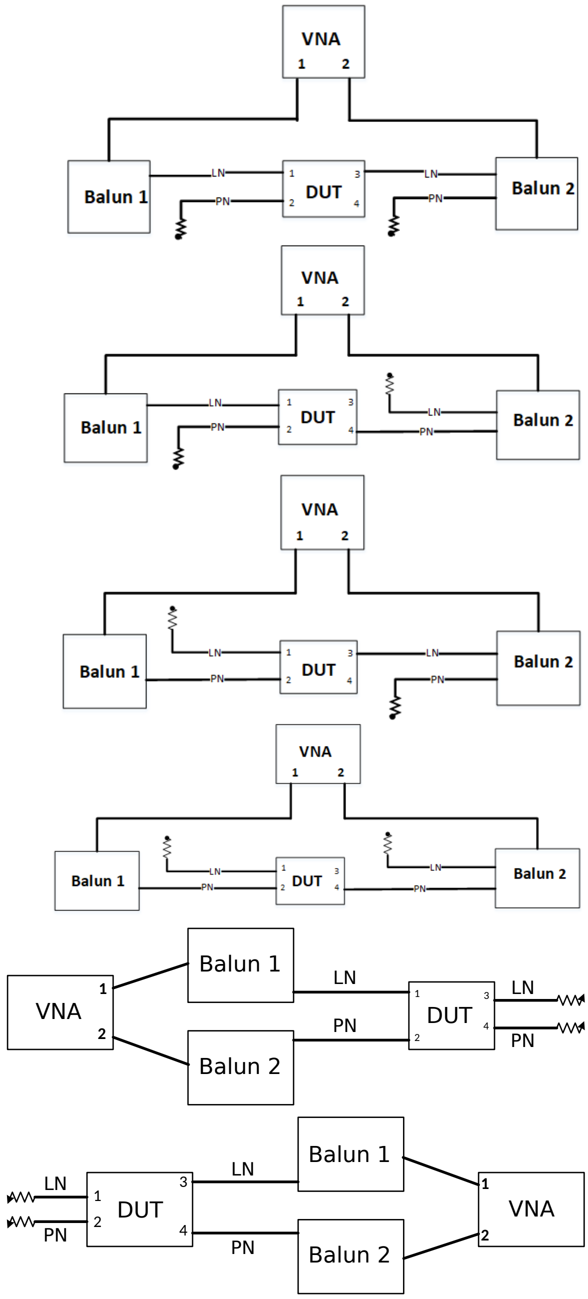

Figure 2: S-parameter measurements for a DUT

[1]For convenience, we omitted ordinate labels.

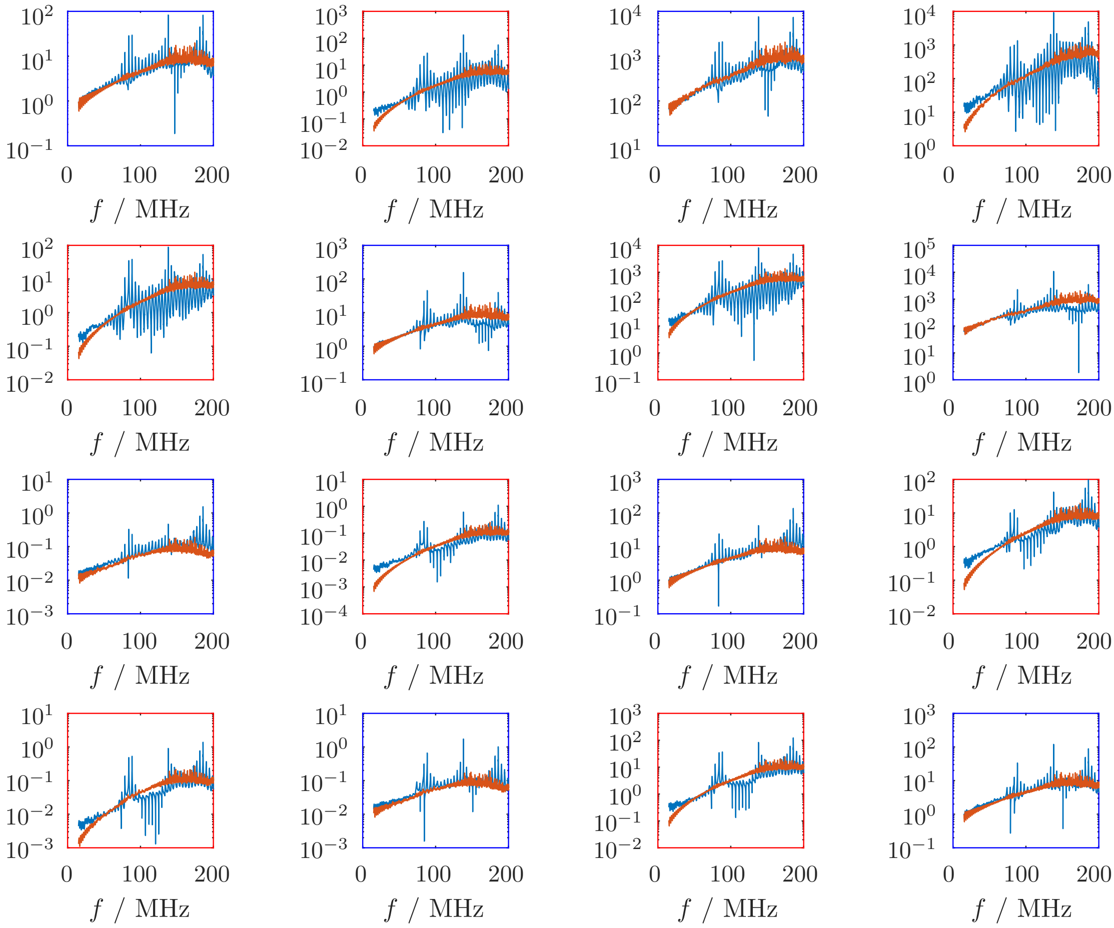

Figure 3: Amplitudes of 4×4 ABCD parameters from scattering parameters and modeled ones

Acknowledgment

This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under HE 3654/27-1.

References

[1] T. Reveyrand, "Multiport conversions between S, Z, Y, H, ABCD, and T parameters," in *2018 International Workshop on Integrated Nonlinear Microwave and Millimetre-wave Circuits (INMMIC)*. IEEE, 2018, pp. 1–3.

[2] "Test procedures for digital subscriber line (DSL) transceivers," G. 996.1.

[3] E. Liu, Y. Gao, G. Samdani, O. Mukhtar, and T. Korhonen, "Broadband characterization of indoor powerline channel and its capacity consideration," in *IEEE International Conference on Communications, 2005. ICC 2005. 2005*, vol. 2, 2005, pp. 901–905.