# Phase-Invariant Coded Phase Shift Keying Using Reed-Muller Codes

Werner Henkel

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To avoid random walks of the carrier loop as response to carrier-phase instabilities, phase-invariant signal space codes for phase shift keying (PSK) are developed. Conditions for the component codes of Zinoviev's Generalized Concatenated Codes (multilevel codes) are derived to achieve phase invariance. Furthermore, conditions are given that additionally guarantee invariance against the proposed differential encoding. It is shown that appropriately chosen Reed-Muller (RM) codes fulfil both requirements. The conditions for phase- and differential invariance become conditions on the order of the RM codes. Some examples with asymptotic coding gains of 3 to 6 dB are presented. Additional outer Reed-Solomon codes further increase this gain.

**Phaseninvariante codierte Phasenumtastung mit Reed-Muller-Codes**

Es werden rotationsinvariante Codes für codierte Phasenumtastung vorgestellt. Diese garantieren, daß Instabilitäten der Trägerphase nicht zu Kurzzeitausfällen der Trägersynchronisation (random walks) führen. Auf der Grundlage des Konstruktionsprinzips der verallgemeinert verketteten Codes nach Zinoviev, im speziellen auch als Mehr-Stufencodes bezeichnet, werden Bedingungen für die Komponentencodes erarbeitet, die für Rotationsinvarianz zu erfüllen sind. Vorschläge zur differentiellen Codierung führen zu Zusatzbedingungen, die ebenfalls eingehalten werden müssen. Reed-Muller-Codes erweisen sich als geeignet, um beiden Bedingungen auf einfache Weise zu genügen. Hieraus folgen Bedingungen für die zu wählende 'Ordnung' der Reed-Muller-Codes. Es werden Beispiele mit asymptotischen Codierungsgewinnen von 3 bis 6 dB angegeben. Eine zusätzliche Verkettung mit Reed-Solomon-Codes erlaubt noch eine Steigerung des Gewinns.

## 1. Introduction

Practical implementations (see e.g. [1]) have shown that Ungerböck's 8-PSK trellis coded modulation (TCM) schemes [2] suffer from phase-stability problems that diminish their nominal coding gain. Cycle slips of the carrier phase result from the small retention range of the phase locked loop, which is only half as wide as that for conventional quadrature phase-shift keying $(-\pi/8, \pi/8)$. Most of the convolutional codes that have been used, exhibit only a $180^o$-phase invariance with a wide random walk zone between adjacent retention ranges (see Fig. 1a). Nonlinear codes with a $90^o$-phase invariance have been found by Ungerböck et al., and multidimensional constructions have been published by Wei [3], Pietrobon et al. Recently, some interesting results based on convolutional codes over rings have been obtained by Massey and his group [4]. To improve phase stability of conventional TCM schemes, additional measures have been developed, e.g., insertion of QPSK signals into the sequence of 8-PSK symbols [5],[6].

In this paper we show that if Reed-Muller codes are combined in a certain way according to the generalized concatenation scheme[1] of Zinoviev [7], [8], [9], then phase-invariant coded phase shift keying automatically results. The phase-invariance property ensures that, after a cycle slip, another stable working point is immediately reached with no random walk (see Fig. 1b).

Before the way, RM codes are assembled in the code construction, is discussed in detail, some aspects of the encoding of set-partitions should be treated.

[1] 'multilevel codes' according to Imai, Calderbank et al.

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Dr. W. Henkel, Research Institute of Deutsche Bundespost Telekom, P. O. Box 10 00 03, D-6100 Darmstadt, Germany.

## 2. Zinoviev's Generalized Concatenated Codes

The signal space codes to be constructed are based on protecting the labeling of the set partitions of the modulation alphabet against errors. The set partitioning for 8-PSK with the corresponding labeling that we shall use is given in Fig. 2.

Note that the labels, considered as the binary representation of integers, are in natural order with increasing phase. Error correcting block codes are applied to the labels according to Zinoviev's construction for generalized concatenated codes (GCC) [7], [8], being aware that there are several other more or less equivalent possibilities for defining the encoding of the set partitions (e.g. [10], [11], [12]).

Due to its practical significance, GCC are explained on the basis of 8-PSK. The generalization to $M$-PSK should be obvious.

According to the definition of GCC, the points of the 8-PSK are to be seen as an *inner code*. Three binary *outer codes* are needed to encode the three partitions of the 8-PSK signal set as shown in Fig. 2, each of the same length $n$. The codeword $A$ of the GCC $\mathcal{A}$ can be

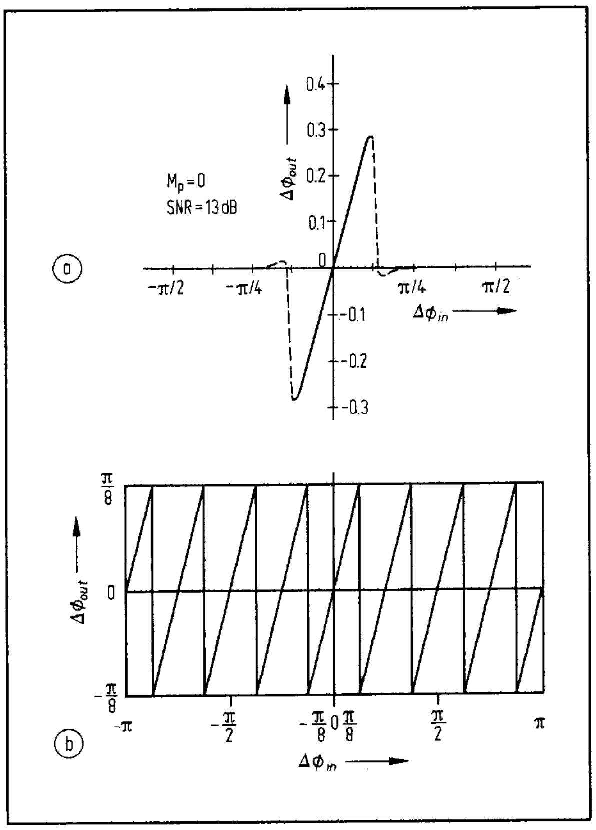


Fig. 1. S-curves of decision directed carrier-phase loops. (a) $180^o$-invariant characteristic based on a 4-state TCM scheme with tentative decisions [2], (b) $45^o$-invariant characteristic (noise-free).

written as a matrix

$$\mathcal{A} \ni A = \begin{pmatrix} a^{(1)} \\ a^{(2)} \\ a^{(3)} \end{pmatrix} = \qquad (1)$$

$$= \begin{pmatrix} a_1^{(1)}, a_2^{(1)}, \ldots, a_n^{(1)} \\ a_1^{(2)}, a_2^{(2)}, \ldots, a_n^{(2)} \\ a_1^{(3)}, a_2^{(3)}, \ldots, a_n^{(3)} \end{pmatrix} \begin{matrix} \longleftarrow \in \mathcal{A}^{(1)} \\ \longleftarrow \in \mathcal{A}^{(2)} . \\ \longleftarrow \in \mathcal{A}^{(3)} \end{matrix}$$

The rows $a^{(1)}$, $a^{(2)}$, and $a^{(3)}$ are codewords of the outer codes $\mathcal{A}^{(1)}$, $\mathcal{A}^{(2)}$, and $\mathcal{A}^{(3)}$, respectively. The columns label the corresponding points of the 8-PSK signal set. The first component determines the 4-PSK subset, the second a 2-PSK subset of the 4-PSK set and the third component decides which point of the 2-PSK set will be taken. Fig. 2 illustrates the procedure. The minimum quadratic Euclidean distance between encoded words of such a scheme is known to be

$$d_{E_{\min}} = \min_j \{ d_H^{(j)} d_E^{(j)} \}, \qquad (2)$$

where $d_H^{(j)}$ is the minimum Hamming distance of the $j$th outer code $\mathcal{A}^{(j)}$ and $d_E^{(j)}$ is the minimum quadratic Euclidean distance between the corresponding $2^{(3-j)}$-PSK subsets.

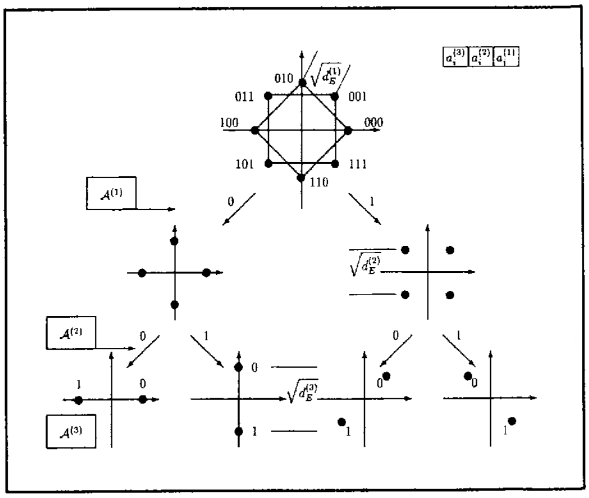


Fig. 2. Set partitions of the 8-PSK.

In the sequel, it will be shown for general $M$-PSK with $M = 2^k$ that the necessary and sufficient conditions for full phase invariance, i.e., invariance with respect to multiples of $2\pi/M$, are:

$$(1, 1, \ldots, 1) \in \mathcal{A}^{(1)} \qquad (3)$$

and for an arbitrary $a^{(j)} \in \mathcal{A}^{(j)}$, $j = 1, \ldots, k$,

$$\prod_{\eta=1}^{j-1} a^{(\eta)} \in \mathcal{A}^{(j)}, \quad j = 2, \ldots, k . \qquad (4)$$

Section 4 shows that these conditions are satisfied by special Reed-Muller codes with $\mathcal{A}^{(1)} \subset \mathcal{A}^{(2)} \subset \ldots \subset \mathcal{A}^{(k)}$, where the inclusions are all strict. In Section 5 more stringent conditions are derived to ensure full phase invariance when differential encoding is used.

Beforehand, the effects of phase shifts on the codewords are described and conditions for phase invariance with respect to multiples of $2\pi/M$ are derived.

## 3. Necessary and Sufficient Conditions for Phase Invariance with Respect to Multiples of $2\pi/M$

A signal space code is rotationally invariant with respect to *multiples* of $2\pi/M$ if and only if it is invariant with respect to a $2\pi/M$ rotation. Therefore, we need to consider only rotation by $2\pi/M$.

The $2\pi/M$ rotation of a single phase symbol is equivalent to the addition of $(r^{(k)}, r^{(k-1)}, \ldots, r^{(1)}) = (0, 0, \ldots, 0, 1)$ to each $(a_i^{(k)}, a_i^{(k-1)}, \ldots, a_i^{(1)})$, $i = 1, \ldots, n$, which again equals the mod-$M$ addition of the all-ones vector to the base $M$ representation (octal for 8-PSK) of $(a_i^{(k)}, \ldots, a_i^{(1)})$.

For illustration, Table 1 shows the addition of $(0, 0, 0, 1) \equiv \pi/8$ to all possible 4-tuples of the 16-PSK.

Table 1. Effects of a phase shift by $\pi/8$, the 'symmetry angle', on the labels of the points of the 16-PSK.

| $+(0,0,0,1),\ +\pi/8$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | ⟶ | 0 | 0 | 0 | 1 |
| 0 | 0 | [0] | 1 | ⟶ | 0 | 0 | [1] | 0 |
| 0 | 0 | 1 | 0 | ⟶ | 0 | 0 | 1 | 1 |
| 0 | [0] | [1] | 1 | ⟶ | 0 | [1] | [0] | 0 |
| 0 | 1 | 0 | 0 | ⟶ | 0 | 1 | 0 | 1 |
| 0 | 1 | [0] | 1 | ⟶ | 0 | 1 | [1] | 0 |
| 0 | 1 | 1 | 0 | ⟶ | 0 | 1 | 1 | 1 |
| [0] | [1] | [1] | 1 | ⟶ | [1] | [0] | [0] | 0 |
| 1 | 0 | 0 | 0 | ⟶ | 1 | 0 | 0 | 1 |
| 1 | 0 | [0] | 1 | ⟶ | 1 | 0 | [1] | 0 |
| 1 | 0 | 1 | 0 | ⟶ | 1 | 0 | 1 | 1 |
| 1 | [0] | [1] | 1 | ⟶ | 1 | [1] | [0] | 0 |
| 1 | 1 | 0 | 0 | ⟶ | 1 | 1 | 0 | 1 |
| 1 | 1 | [0] | 1 | ⟶ | 1 | 1 | [1] | 0 |
| 1 | 1 | 1 | 0 | ⟶ | 1 | 1 | 1 | 1 |
| [1] | [1] | [1] | 1 | ⟶ | [0] | [0] | [0] | 0 |

We observe that in general the binary numbering of the phase label is changed according to

$$a_i^{(1)}|_{+2\pi/M} = a_i^{(1)} + 1 \mod 2 \qquad (5)$$

$$a_i^{(j)}|_{+2\pi/M} = a_i^{(j)} + \prod_{\eta=1}^{j-1} a_i^{(\eta)} \mod 2, \qquad (6)$$

$$j = 2, \ldots, k\,.$$

Together with the assumption of linearity of the outer codes, it follows that the necessary and sufficient conditions for phase-invariant coded modulation based on Zinoviev's scheme are

$$(1,1,...,1) \in \mathcal{A}^{(1)}, \qquad (7)$$

$$\prod_{m=1}^{j-1} a^{(m)} \in \mathcal{A}^{(j)}, \quad j = 2, \ldots, k$$

The following section shows that Reed-Muller codes as outer codes fulfil these conditions if they are chosen in a special *increasing order*.

## 4. Combining Reed-Muller Codes to Form Phase-Invariant PSK

An $r$th order Reed-Muller code RM($r, m$) of block length $2^m$ can be defined by a block generator matrix of the form

$$G = \begin{pmatrix} G_0 \\ G_1 \\ \vdots \\ G_r \end{pmatrix}, \qquad (8)$$

where $G_0$ is the all-ones vector of length $2^m$. $G_1$ is an $m \times 2^m$-matrix, consisting of each binary $m$-tuple appearing once as a column. $G_l$ ($2 \le l \le r$) is formed by all different products of $l$ rows of $G_1$. The number of information bits is thus

$$k = 1 + \binom{m}{1} + \ldots + \binom{m}{r}\,. \qquad (9)$$

The minimum Hamming distance can be shown to be $d_H = 2^{m-r}$ ([13], p.60).

For illustration, the elements of the generator matrix for the 3rd order RM code of length $2^4 = 16$ are given by

$$\begin{aligned}
G_0 &= [1111111111111111] = [c_0] \\
G_1 &= \begin{bmatrix} 0000000011111111 \\ 0000111100001111 \\ 0011001100110011 \\ 0101010101010101 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \\
G_2 &= \begin{bmatrix} 0000000000001111 \\ 0000000000110011 \\ 0000000001010101 \\ 0000001100000011 \\ 0000010100000101 \\ 0001000100010001 \end{bmatrix} = \begin{bmatrix} c_1c_2 \\ c_1c_3 \\ c_1c_4 \\ c_2c_3 \\ c_2c_4 \\ c_3c_4 \end{bmatrix}\,. \qquad (10) \\
G_3 &= \begin{bmatrix} 0000000000000011 \\ 0000000000000101 \\ 0000000000010001 \\ 0000000100000001 \end{bmatrix} = \begin{bmatrix} c_1c_2c_3 \\ c_1c_2c_4 \\ c_1c_3c_4 \\ c_2c_3c_4 \end{bmatrix}
\end{aligned}$$

Obviously, $\mathrm{RM}(0,m) \subset \mathrm{RM}(1,m) \subset \mathrm{RM}(2,m) \subset \ldots \subset \mathrm{RM}(m,m)$, where all inclusions are strict. Moreover,

$$(1,1,\ldots,1) \in \mathrm{RM}(r,m)\,, \qquad (11)$$

$$a_{r^{(\eta)}} \in \mathrm{RM}(r^{(\eta)},m) \Longrightarrow \prod_{\sum_{\eta=1}^{j-1} r^{(\eta)} \le r^{(j)}} a_{r^{(\eta)}} \in \mathrm{RM}(r^{(j)},m)\,.$$

It follows that phase-invariant coded $M$-PSK ($M = 2^k$) is achieved if the outer codes $\mathcal{A}^{(j)}$ are chosen according to

$$\mathcal{A}^{(1)} = \mathrm{RM}(r^{(1)},m),\ \ldots,\ \mathcal{A}^{(k)} = \mathrm{RM}(r^{(k)},m),$$

$$r^{(1)} < r^{(2)} < \ldots < r^{(k)}, \qquad (12)$$

$$\sum_{\eta=1}^{j-1} r^{(\eta)} \le r^{(j)} \vee r^{(j)} = m\,, \quad j = 2,\ldots,k\,,$$

which in turn means that $\mathcal{A}^{(1)} \subset \mathcal{A}^{(2)} \subset \ldots \subset \mathcal{A}^{(k)}$, equality being excluded.

In Table 2 some GCC schemes with RM codes for coded 8-PSK, fulfilling these conditions, are given.

The 3-dB gain of the first example corresponds to what is achieved with 4-state trellis codes, and the 6-dB gains of the other examples are comparable to that achievable with 128-state TCM.

Table 2. Phase invariant multilevel coding schemes with Reed-Muller (RM) codes. ($d_{E_{\min}}/2\,/\mathrm{dB} = 10\lg \min_j\{(d_H^{(j)} \cdot d_E^{(j)})/2\}$, where 2 is the quadratic Euclidean distance of the uncoded 4-PSK, $R$: coderate).

| $(j)$ | $(n,k,d_H)$ | RM$(r,m)$ | $d_E$ | $d_H d_E$ | $d_{E_{\min}}/2\,/\mathrm{dB}$ | $R = \Sigma k^{(j)}/\Sigma n^{(j)}$ |
|---|---|---|---|---|---|---|
| 1 | (8,1,8) | (0,3) | 0.586 | 4.688 | | |
| 2 | (8,7,2) | (2,3) | 2 | 4 | 3 | 2/3 |
| 3 | (8,8,1) | (3,3) | 4 | 4 | | |
| 1 | (16,1,16) | (0,4) | 0.586 | 9.376 | | |
| 2 | (16,11,4) | (2,4) | 2 | 8 | 6 | $9/16 < 2/3$ |
| 3 | (16,15,2) | (3,4) | 4 | 8 | | |
| 1 | (32,6,16) | (1,5) | 0.586 | 9.376 | | |
| 2 | (32,26,4) | (3,5) | 2 | 8 | 6 | $21/32 \approx 2/3$ |
| 3 | (32,31,2) | (4,5) | 4 | 8 | | |
| 1 | (64,22,16) | (2,6) | 0.586 | 9.376 | | |
| 2 | (64,57,4) | (4,6) | 2 | 8 | 6 | $0.74 > 2/3$ |
| 3 | (64,63,2) | (5,6) | 4 | 8 | | |

Having shown that GCC schemes with RM codes yield fully phase-invariant coded $M$-PSK, it remains to describe the necessary differential encoding and decoding techniques.

## 5. Differential Coding for Phase-Invariant GCC Schemes

Phase-invariant coded modulation makes sense only if a differential encoding and corresponding decoding is carried out. The data is then embodied by differences (in phase) and is thus not corrupted by any phase offset.

First, the 8-PSK as special case of the $M$-PSK is considered. Generalizations to $M$-PSK are straightforward.

The differential coding proposed here is similar to the one by Oerder and Meyr in [14]. It consists of a differential encoding mod 8 over a modulation interval of block length $n$ after the GCC encoder (see Fig. 3). The differential decoder itself has to be placed after the GCC decoder (see Fig. 4), avoiding a 3-dB loss. Thus, the GCC schemes additionally must be invariant against differential encoding mod 8.

In order to show that this requirement can be fulfilled by RM codes, too, we consider the addition of two GCC codewords in their octal representation. If the set of GCC codewords with RM codes turns out to be closed under addition, the differential encoding yields a valid codeword, thus allowing the differential decoding to be performed after GCC decoding.

Let $A_1 \in \mathcal{A}$, $A_2 \in \mathcal{A}$, $A_1 \neq A_2$. The addition of $A_1$ and $A_2$ is given by

$$4(A_1^{(3)}+A_2^{(3)})+2(A_1^{(2)}+A_2^{(2)})+(A_1^{(1)}+A_2^{(1)}), \quad (13)$$

where $A_k^{(j)}$ denotes the $j$th binary component of the

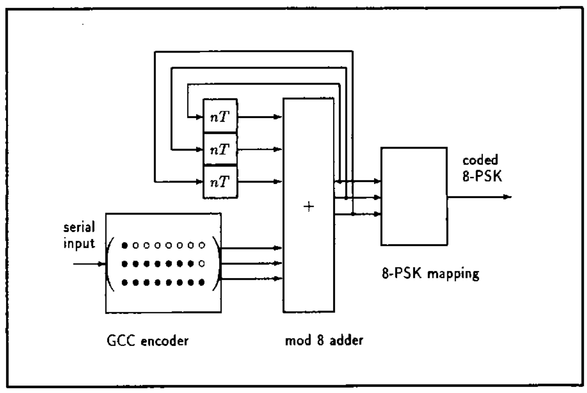


Fig. 3. Differential encoding (8-PSK).

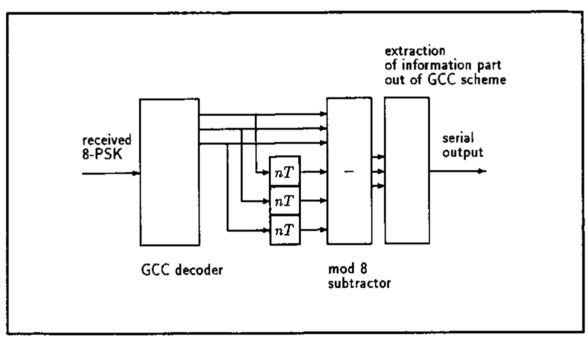


Fig. 4. Differential decoding (8-PSK).

octal number. First we realize that, of course, $(A_1^{(1)} + A_2^{(1)}) \in \mathcal{A}^{(1)}$ mod 2 (linearity). A carry occurs at positions where both $A_1^{(1)}$ and $A_2^{(1)}$ have ones or, equivalently, where the componentwise product $A_1^{(1)} A_2^{(1)}$ yields ones. Thus, products of codewords out of $\mathcal{A}^{(1)}$

have to be in $\mathcal{A}^{(2)}$.
Now, let $A_3^{(2)} := A_1^{(1)}A_2^{(1)}$. A carry to the third binary component of the octal number occurs if components of $A_1^{(2)} + A_2^{(2)} + A_3^{(2)}$ are greater or equal 2. This in turn means that it occurs at positions where at least two of the components of $A_1^{(2)}$, $A_2^{(2)}$, $A_3^{(2)}$ are one. Thus, $A_1^{(2)}A_2^{(2)} \vee A_1^{(2)}A_3^{(2)} \vee A_2^{(2)}A_3^{(2)}$ has to be in $\mathcal{A}^{(3)}$. Considering that $a \vee b = a + b + ab$, we get

$$A_1^{(2)}A_2^{(2)} \vee A_1^{(2)}A_3^{(2)} \vee A_2^{(2)}A_3^{(2)} = \\ = A_1^{(2)}A_2^{(2)} + A_1^{(2)}A_3^{(2)} + A_2^{(2)}A_3^{(2)}. \quad (14)$$

From this, we deduce that an invariance against differential coding in the general case of $M$-PSK is obtained if

$$A_1^{(j)} \in \mathcal{A}^{(j)},\ A_2^{(j)} \in \mathcal{A}^{(j)} \Longrightarrow A_1^{(j)}A_2^{(j)} \in \mathcal{A}^{(j+1)}, \\ j = 1, \ldots, k-1 \quad (15)$$

In the case of RM$(r, m)$ codes as outer codes this means:

$$2r^{(j)} \leq r^{(j+1)} \quad \vee \quad r^{(j+1)} = m \quad (16)$$

We realize that only the first of the examples given above, consisting of the codes: repetition (8,1,8), parity check (8,7,2), and 'uncoded' (8,8,1), fulfills these more stringent conditions. Other constructions that are phase *and* 'differentially' invariant are given in Table 3.

The schemes labelled with a '∘' are unequal error protection codes and those denoted with a '•' have rates greater than 2/3, enabling to provide additional outer codes (e.g. Reed-Solomon codes). This improves the coding gain by decreasing the overall coderate. The redundancy of the outer code may be chosen such that the original data rate of the uncoded transmission is retained ($R_{\text{total}} = 2/3$ for coded 8-PSK).

Fortunately, the differential encoder can be modified to ensure that the examples in the first table remain valid, too. Especially the last code scheme with $n = 64$ is of some importance, because it offers a constant 6 dB gain and additionally a rate greater than 2/3 (enables outer RS code). The principle of the encoder structure is given in Fig. 5. The second condition caused by the differential encoding need not be fulfilled for the last component code, because the corresponding coder is positioned after the mod-8 differential encoder.

## 6. Conclusions

Conditions for phase-invariant coded phase-shift keying based on Zinoviev's Generalized Concatenated Codes have been derived. The proposed differential encoding lead to further conditions. Reed-Muller codes as outer codes in Zinoviev's scheme proved to be very well suited to fulfil both requirements. Hereto, the orders $r^{(j)}$ of the RM$(r^{(j)}, m)$ codes have to be chosen according to:

$$\sum_{\eta=1}^{j-1} r^{(\eta)} \leq r^{(j)} \ \vee \ r^{(j)} = m\,,$$

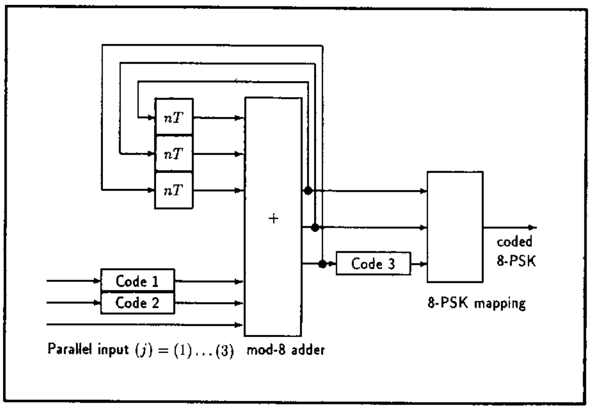


Fig. 5. Modified differential encoding.

$$2r^{(j)} \leq r^{(j+1)} \ \vee \ r^{(j+1)} = m\,.$$

The differential encoder has been modified to free the last component code from the second condition. Examples for coded 8-PSK have been given. Some have unequal error protection capability with asymptotic gains of 3 or 6 dB, respectively. Others have gains of constant 3 or 6 dB, and high coderate, enabling to provide additional outer codes. This leads to a further increase in coding gain.
Finally, it should be pointed out that especially RM codes are favourable, because of their simple and fast decoding.

*Some remarks and comparison with the recently published paper by Kasami et al. [15]*

Most parts of this contribution, excluding the modified differential encoder, have been derived long before the publication of Kasami's paper. The late submission of this paper to AEÜ was reasoned by circumstances that were beyond the authors control. However, parallel results do only belong to the topic of 'phase-invariance' conditions. No differential encoding is treated in the paper by Kasami. Furthermore, the conditions for phase invariance itself are derived in a different way that seems to be rather complicated, compared with this presentation.

Some results of this paper will also be published at [16] and a treatment of rotational invariance of coded QAM can be found in [17].

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Table 3. Phase invariant multilevel coding schemes with Reed-Muller (RM) codes that fulfil additional conditions caused by the proposed differential coding.

| $(j)$ | $(n, k, d_H)$ | RM$(r, m)$ | $d_E$ | $d_H d_E$ | $d_H d_E/2$ /dB | $R = \Sigma k^{(j)}/\Sigma n^{(j)}$ |
|---|---|---|---|---|---|---|
| 1 | (8, 1, 8) | (0,3) | 0.586 | 4.688 | 3.7 | |
| 2 | (8, 7, 2) | (2,3) | 2 | 4 | 3 | 2/3 |
| 3 | (8, 8, 1) | (3,3) | 4 | 4 | 3 | |
| 1 | (16, 1, 16) | (0,4) | 0.586 | 9.376 | 6.71 | |
| 2 | (16, 11, 4) | (2,4) | 2 | 8 | 6 | $7/12 = 0.583 < 2/3$ ○ |
| 3 | (16, 16, 1) | (4,4) | 4 | 4 | 3 | |
| 1 | (16, 5, 8) | (1,4) | 0.586 | 4.688 | 3.7 | |
| 2 | (16, 15, 2) | (3,4) | 2 | 4 | 3 | $3/4 = 0.75 > 2/3$ • |
| 3 | (16, 16, 1) | (4,4) | 4 | 4 | 3 | |
| 1 | (32, 6, 16) | (1,5) | 0.586 | 9.376 | 6.71 | |
| 2 | (32, 26, 4) | (3,5) | 2 | 8 | 6 | 2/3 ○ |
| 3 | (32, 32, 1) | (5,5) | 4 | 4 | 3 | |
| 1 | (32, 16, 8) | (2,5) | 0.586 | 4.688 | 3.7 | |
| 2 | (32, 31, 2) | (4,5) | 2 | 4 | 3 | $79/96 = 0.823 > 2/3$ • |
| 3 | (32, 32, 1) | (5,5) | 4 | 4 | 3 | |

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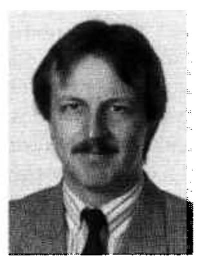

**Werner Henkel** was born in Gelnhausen, Germany, on April 27, 1960. He studied Electrical Engineering (telecommunications) at the Technical University Darmstadt, Germany, where he received his Diploma Degree in 1984. From 1984 to 1989 he had a contract as a research assistant at the same institution and received his Doctoral Degree in 1989. His thesis was on coding with complex numbers. During this time, he also held lectures at the Fachhochschule Frankfurt.

In 1989 he joined the research institute of the Deutsche Bundespost Telekom, at Darmstadt. His work concentrates on digital communications, especially coding, coded modulation, synchronization, equalizing and neural networks. He also held internal courses on information theory and coding.

Publications are on analog codes, coding and interpolation, Toeplitz algorithms, coded modulation, synchronization, and A/D-conversion.