Another Application for Trellis Shaping: PAR Reduction for DMT (OFDM) — unpublished appendix —

Werner Henkel, *Member, IEEE*, and Björn Wagner

Appendix

A. Single-carrier PAR

The mean power of an M-QAM constellation has already been given in Eq. (1). Let us denote this as $\bar{P}_{M-QAM}$. The peak power of one carrier is

$$\hat{P}_{M-QAM} = \frac{a^2}{2} \cdot (\sqrt{M}-1)^2 \, . \tag{1}$$

Under conjugacy constraints and $F_0 = F_{\frac{N}{2}} = 0$, we obtain the time-domain components

$$\begin{aligned} f_l &= \sum_{m=0}^{N-1} F_m \, e^{j\,\frac{2\pi}{N}\,l\,m} \\ &= \sum_{m=1}^{\frac{N}{2}-1} \left(F_m \, e^{j\,\frac{2\pi}{N}\,l\,m} + F_{N-m} \, e^{j\,\frac{2\pi}{N}\,l\,(N-m)} \right) \\ &= \sum_{m=1}^{\frac{N}{2}-1} \left(|F_m| \; e^{j\,(\frac{2\pi}{N}\,l\,m+\varphi_m)} + |F_m| \; e^{-j\,(\frac{2\pi}{N}\,l\,m+\varphi_m)} \right) \\ &= \sum_{m=1}^{\frac{N}{2}-1} 2\,|F_m| \cos\!\left(\frac{2\pi}{N}\,l\,m + \varphi_m \right) . \end{aligned} \tag{2}$$

With some trivial operations, the mean power in time domain can be obtained as

$$\bar{P} = \sum_{m=1}^{\frac{N}{2}-1} 2\,|F_m|^2 \, . \tag{3}$$

Considering only one conjugate pair, i.e., one real QAM time-domain signal, we get

$$\overline{P}_{2\,conj.\,carr.} = \frac{a^2}{3} \cdot (M-1) \, , \tag{4}$$

$$\begin{aligned} \hat{P}_{2\,conj.\,carr.} &= 4\,|F_{max}|^2 \\ &= 2\,a^2 \cdot (\sqrt{M}-1)^2 \, , \end{aligned} \tag{5}$$

which leads to the PAR of the single-carrier QAM

$$PAR_{2\,conj.\,carr.} = 6 \cdot \frac{(\sqrt{M}-1)}{(\sqrt{M}+1)} \, . \tag{6}$$

The limit of the PAR for $M \to \infty$ is 6, which is also obtained when deriving the PAR for a continuous approximation. The derivation for such a constant density over a square $(A \times A)$ is given subsequently.

$$\begin{aligned} \bar{P}_{cont.} &= \left(\frac{1}{2A}\right)^2 \int_{-A}^{A}\int_{-A}^{A} (x^2+y^2)\,dx\,dy = \frac{2}{3}\,A^2 \\ \hat{P}_{cont.} &= 2A^2 \end{aligned}$$

$$\overline{P}_{2\,conj.\,carr.} = 2 \cdot \overline{P}_{cont.} \text{ and } \hat{P}_{2\,conj.\,carr} = 4 \cdot \hat{P}_{cont.}$$

$$PAR_{cont.} = \lim_{M\to\infty} PAR_{2\,conj.\,carr.} = 6$$

B. PAR for rectangular density

Let the density be rectangular with

$$p(x) = \begin{cases} 1/(2\hat{s}) \, , & -\hat{s} < x \le \hat{s} \\ 0 \, , & \text{else} \end{cases} \, . \tag{7}$$

The average power is

$$\bar{P} = \frac{1}{2\hat{s}} \cdot 2 \cdot \int_0^{\hat{s}} x^2 dx = \frac{\hat{s}^2}{3} \, . \tag{8}$$

The PAR results to be

$$PAR = \frac{\hat{s}^2}{\hat{s}^2/3} = 3 \, . \tag{9}$$