Check-Irregular LDPC Codes for Unequal Error Protection Under Iterative Decoding

Lucile Sassatelli, Werner Henkel, David Declercq

4th May 2006

Outline

Outline

Problem

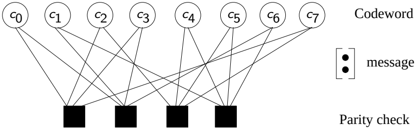
- **Context**:
  Unequal Error Protection (UEP): for transmission of multi-media content with heterogeneous sensibility to errors.
  LDPC codes irregularity adaptable to UEP.
- **Goal**:
  Creating flexible UEP coding scheme based on LDPC codes to process different kind of scalable data by the same system.
- **Approach**:
  Adapting the check node profile of bit-regular LDPC codes to speed up the convergence of the most protected bits.

Outline

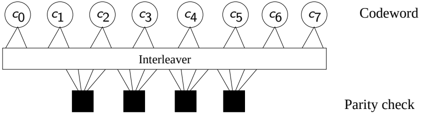
LDPC Codes over $GF(2)$: The Binary Case

- Binary LDPC codes: Linear application from $GF(2)^K$ to $GF(2)^N$.
- factor graph = non oriented bayesian graph

$$H=\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

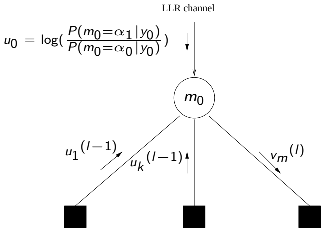


Family $(d_v = 2, d_c = 4)$



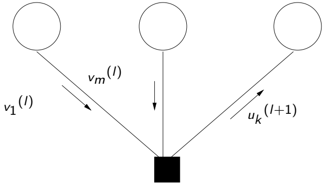
Decoding of LDPC Codes: The Belief Propagation Algorithm

- Update of a degree d_v bit node



$$v_m^{(l)} = u_0 + \sum_{k=1,k\neq m}^{d_v} u_k^{(l-1)}, \forall m = 1 \ldots d_v$$

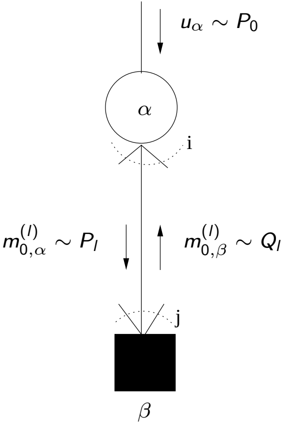
- Update of a degree d_c check node



$$\tanh \frac{u_k^{(l)}}{2} = \Pi_{m=1,m\neq k}^{d_c} \tanh \frac{v_m^{(l)}}{2}$$

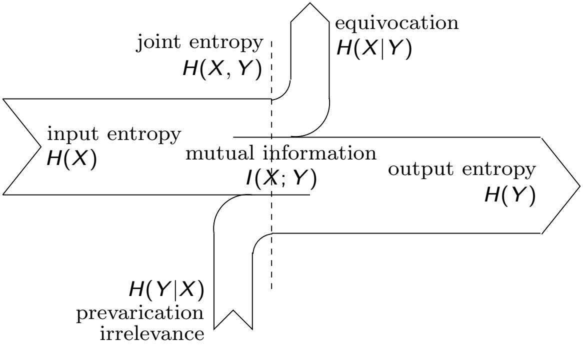
Outline

A Useful Tool: The Density Evolution



- To track the probability densities of messages on edges along the decoding process
- Allows to know the a posteriori density of messages after converging to a fixed point
  $\rightarrow$ Expression of the error probability

Mutual Information Between a Message and the Input of the Channel



Density Evolution with a Gaussian Approximation: Evolution of the Mutual Information

Goal: Practical search of the best codes family (λ, ρ)

- What GA does: Projection of message densities on a gaussian kernel.
- Symmetry of messages assumed to be gaussian $\rightarrow$ $u^t \sim \mathcal{N}(m^t, 2m^t)$ Only one parameter charaterizes the family.

Therefore approximative but easy optimization of binary LDPC codes.

Outline

Our Cost Function

The cost function is the relative speed of convergence of a given class of protection compared with the speed of the whole codeword.

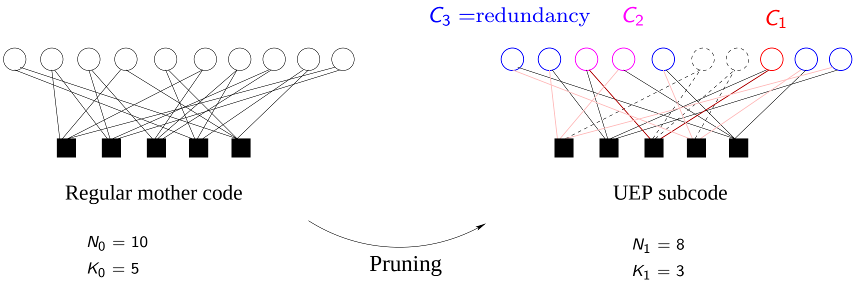
$$x_{cv}^{(l)^{(C_k)}} - x_{cv}^{(l-1)} \geq 1 - J\left((\sum_{d \in C_k} \rho^{(C_k)}(d)d - 1) J^{-1}(1 - x_{vc}^{(l)}) \right) - x_{cv}^{(l-1)}$$

The average check connection degree of the class C_k:

$$\overline{\rho}^{(C_k)} = \sum_{d=d_{min}^{(C_k)}}^{d_{max}^{(C_k)}} \rho^{(C_k)}(d)d$$

has to be minimized.

Pruning of a Bit-Regular Mother Code

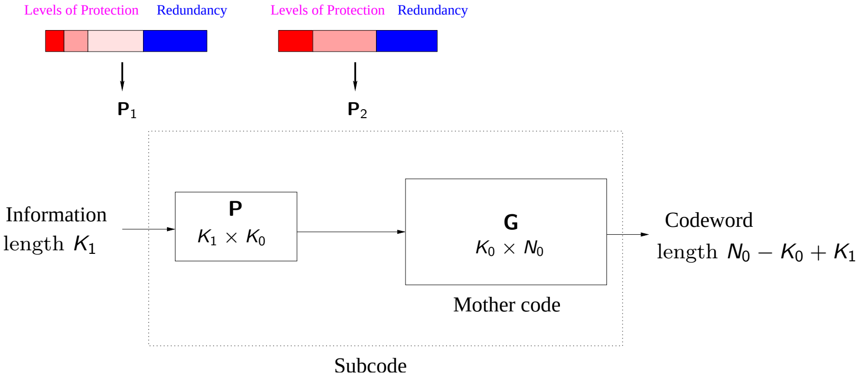


$N_0 = 10$
$K_0 = 5$
$R_0 = \frac{1}{2}$

$N_1 = 8$
$K_1 = 3$
$R_1 = \frac{3}{8}$

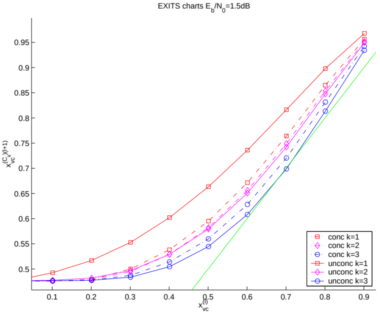
Flexible Design of UEP LDPC Codes

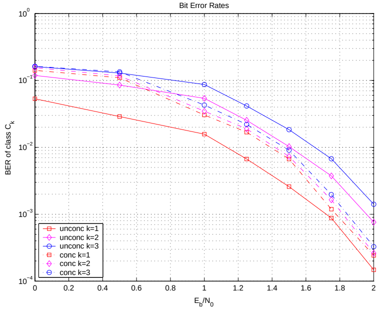
- **P** is optimized with iterative pruning of a mother code



Outline

Results obtained by pruning a (3,6) LDPC code





Conclusion

- We have optimized the check-irregularity of a bit-regular LDPC code to speed up the local convergence of messages, thereby creating UEP behavior.
- We implemented the cost function by a highly flexible pruning method, that allows to have different UEP configurations with a same mother code.
- The next step of this work would be to combine bit and check irregularities to provide best unequal error protection.