Communication Theory

Letter

A Simple Multilevel Block-Coding Scheme with $\pi/4$-QPSK Properties

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Abstract. A multilevel block-coded QPSK with a similar envelope and trajectories as $\pi/4$-QPSK is proposed. Additionally, it is rotationally invariant and, thus, allows coherent detection. The asymptotic coding gain is 3 dB, whereas the effective coding gain at a bit-error probability of 10^{-6} is around 2 dB against uncoded $\pi/4$-QPSK. The corresponding trellis of a full maximum-likelihood decoder has only four states, which results in a very low complexity.

1. INTRODUCTION

Several countermeasures to reduce envelope fluctuations of the modulated carrier leading to increased spectral sidelobes after nonlinear high power amplification have been proposed. Widely known are the group of CPM (Continuous Phase Modulation) methods which have a constant envelope. However, practical implementations are usually restricted to the binary case with the modulation index $h = 1/2$, called MSK (Minimum Shift Keying), or variations thereof like GMSK (Gaussian MSK). The spectral efficiency of these modulation schemes is relatively low.

Another alternative has first ([1]) been proposed by Baker in [1] and was extensively examined by Feher [2, 3]. The solution was denoted by $\pi/4$-QPSK, which stands for a QPSK with a phase offset alternating between zero and $\pi/4$ every modulation symbol. This linear modulation technique, at least, avoids transitions through the origin of the complex plane, which otherwise would have induced a complete envelope breakdown in the bandlimited transmitted signal. The method yields the same bit-error performance as conventional QPSK under Gaussian noise (AWGN) and is known to be only slightly inferior to it in the presence of (linear or nonlinear) intersymbol interference. $\pi/4$-QPSK will be applied in the second generation of cellular mobile radio systems in the USA and is proposed for the Japanese system standard [3].

Similar trajectories have also been achieved by Morrison [4], who applied Forney's trellis shaping [5] by using another metric, which is some sort of 'weight' of the transitions. Unfortunately, it implies a doubling of the modulation alphabet when transmitting with the same bit rate. This means that, e.g., 8-PSK must be used instead of 4-PSK, which for practical reasons is usually not acceptable (synchronization aspects). Furthermore, the receiver has to make decisions on an alphabet of larger size with decreased Euclidean distance between adjacent symbols, which drastically deteriorates the error performance. A 'shaping' without a doubling of the alphabet size is currently investigated by the authors, but is not treated here.

This paper shows a block-coded $\pi/4$-QPSK, where the $\pi/4$ shifts directly result from the selection of one of the block codes. The proposal is based on so-called multilevel codes [6, 7] or Generalized Concatenated Codes (GCC) [8, 9, 10]. Multilevel codes are shortly described in the following section. Then the coded $\pi/4$-QPSK is presented. Simulation results and a short summary conclude this contribution.

2. MULTILEVEL CODED 8-PSK

Multilevel codes protect the labeling of the set partitions of the modulation alphabet against errors. The set partitioning for 8-PSK with the corresponding labeling that we shall use is given in Fig. 1. Note that the labels considered as the binary representation of integers are in natural order with increasing phase.

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([1]) As far as the authors know.

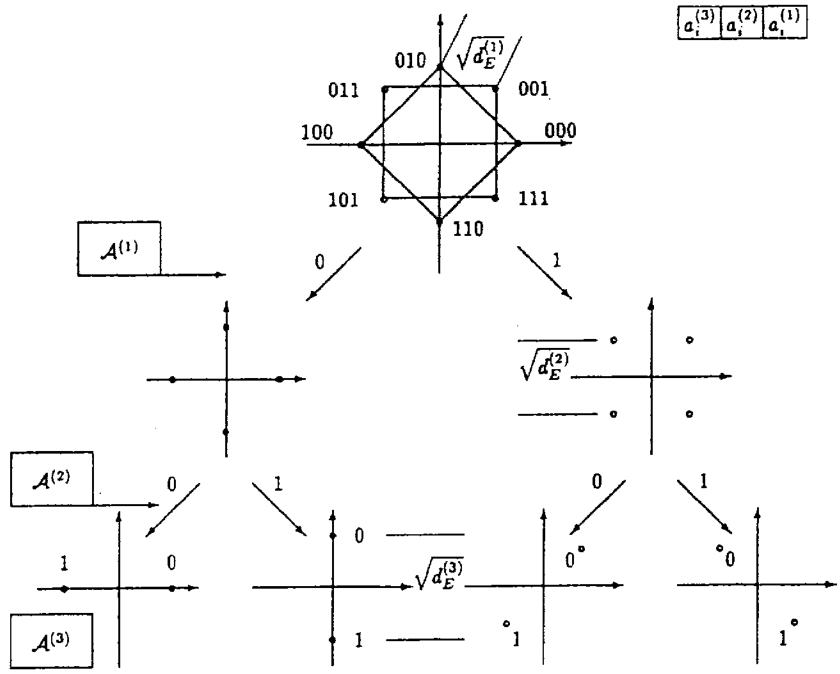

Fig. 1 - Set partitions of the 8-PSK.

Three binary component codes are needed to encode the three partitions of the 8-PSK signal set as shown in Fig. 1, for convenience, each of the same length n. The codeword A of the multilevel code $\mathcal{A}$ can be written as a matrix

$$\mathcal{A} \ni A = \begin{pmatrix} A^{(1)} \\ A^{(2)} \\ A^{(3)} \end{pmatrix} = \begin{pmatrix} a_1^{(1)}, a_2^{(1)}, \dots, a_n^{(1)} \\ a_1^{(2)}, a_2^{(2)}, \dots, a_n^{(2)} \\ a_1^{(3)}, a_2^{(3)}, \dots, a_n^{(3)} \end{pmatrix} \begin{matrix} \leftarrow \in \mathcal{A}^{(1)} \\ \leftarrow \in \mathcal{A}^{(2)} \\ \leftarrow \in \mathcal{A}^{(3)} \end{matrix} \qquad (1)$$

The rows $A^{(1)}$, $A^{(2)}$, and $A^{(3)}$ are codewords of the component (or outer(2)) codes $\mathcal{A}^{(1)}$, $\mathcal{A}^{(2)}$, and $\mathcal{A}^{(3)}$, respectively. The columns label the corresponding points of the 8-PSK signal set. The components of the first 'outer' codeword determine the 4-PSK subset, the ones of the second codeword a 2-PSK subset of the 4-PSK set and the ones of the third codeword decide, which point of the 2-PSK set will be taken. Fig. 1 illustrates this procedure. The minimum quadratic Euclidean distance between encoded words of such a scheme is known to be (see, e.g., [10], p. 591)

$$d_{E_{\min}} = \min_j \left\{ d_H^{(j)} \cdot d_E^{(j)} \right\} \qquad (2)$$

where $d_H^{(j)}$ is the minimum Hamming distance of the j-th outer code $\mathcal{A}^{(j)}$ and $d_E^{(j)}$ is the minimum quadratic Euclidean distance between the corresponding $2^{(3-j)}$-PSK subsets. (The preceding description has been quoted from [11]; see also [14, 15]).

In the following section we shall show that choosing special component codes leads to a modulation that is very similar to ordinary $\pi/4$-QPSK.

3. SCHEME FOR CODED '$\pi/4$-QPSK'

The proposed scheme actually is exactly equivalent to a $\pi/4$-QPSK scheme within code blocks A of length n, where n has to be an integral multiple of 4, as will be explained later. Between successive code blocks the regular alternation of the two QPSK subsets is allowed to be interrupted, depending on the transmitted data.

The multilevel coding scheme consists of the component codes:

1) Coset of the Repetition Code ($\mathcal{R}$) – $-\mathcal{R} + (0101010\dots1) = \{(01010\dots1), (10101\dots0)\}$
2) Parity Check Code ($\mathcal{P}$) (even parity)
3) Uncoded ($\mathcal{U}$).

Noting that the Hamming distances are $d_H^{(1)} = n$, $d_H^{(2)} = 2$, $d_H^{(3)} = 1$, and the corresponding squared Euclidean distances are $d_E^{(1)} = 0.586$, $d_E^{(2)} = 2$, $d_E^{(3)} = 4$, we obtain from (2) $d_{E_{\min}} = 4$, if $n \geq 7$. This corresponds to an asymptotic coding gain of 3 dB for additive white Gaussian noise ($10 \log_{10} d_{E_{\min}} / d_{E_{QPSK}} = 10 \log_{10} 4/2$, assuming the same power for coded and uncoded modulation).

The choice of the component codes complies with two requirements. The first is the succession of QPSK

(2) In the sense of Zinoviev's Generalized Concatenated Codes.

subsets within a codeword. This is met by the coset of the repetition code as the first component code, whose components select the QPSK subset from the 8-PSK set. Furthermore, rotational invariance is guaranteed. This is necessary, if coherent demodulation is intended. Note that noncoherent detection (differential detection) leads to a loss of around 3 dB, depending on the channel statistics.

According to [11], the addition of the all-ones word must yield another valid codeword of Code (1). Furthermore, valid codewords of Code (1) have also to be valid for Code (2), assuming linearity of Code (2). For the last code being uncoded, its invariance conditions are always fulfilled.

Regarding the chosen component codes, we observe that, of course, the all-ones word simply interchanges the two codewords of Code (1) when added. Furthermore, the parity of the second code is chosen according to the parity of the first code, which is even if its length is an integral multiple of 4.

So far, we have only proved rotational invariance properties for the code, but not for the information part itself. Applying the differential en- and decoding proposed in [11] (see Figs. 2, 3) additional conditions have to be fulfilled, namely

$$A_1^{(j)} \in \mathcal{A}^{(j)},\ A_2^{(j)} \in \mathcal{A}^{(j)} \Rightarrow A_1^{(j)} \cdot A_2^{(j)} \in \mathcal{A}^{(j+1)},$$

$$j = 1, \ldots, k-1 \qquad (3)$$

For the proposed π/4-QPSK scheme this means that products (logical and) of two arbitrary codewords from Code (1) have to be in Code (2). There are only three possible products which are the codewords themselves and the zero word. It has already been guaranteed that the codewords of Code (1) are valid codewords of Code (2).

For rotational invariance no further conditions have to be considered. We obtain a simple 3dB coding scheme with envelope fluctuation properties similar to π/4-QPSK and a π/4-phase invariance.

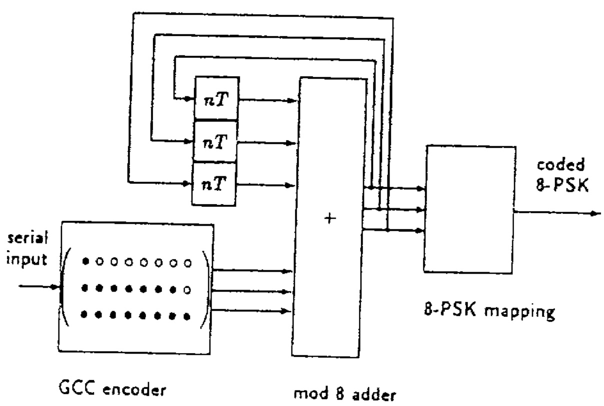

Fig. 2 - Differential encoding (8-PSK).

When applying the differential encoder according to Fig. 2, the desired coset of the repetition code has to be provided by using the regular repetition code ((0 0 . . . 0), (1 1 . . . 1)) in the GCC encoder and initializing the modulo-8 differential encoder with a $3 \times n$ block, the first row being the coset leader (0101 . . . 01) whereas the other positions are set to zero. In Fig. 3 decoding is done according to the coset of the repetition code, whereas after the modulo-8 subtraction the words from the regular code are restored. Due to the differential encoding with modulo-8 addition, $n/2$ carries from codeword $A^{(1)}$ to codeword $A^{(2)}$ occur, if the codeword of the regular repetition code is the all-ones word (1 1 . . . 1). Therefore, the parity of codeword $A^{(2)}$ is changed $n/2$ times. However, if $n/2$ is even, i.e., n is an integral multiple of 4, the even parity of $A^{(2)}$ is preserved.

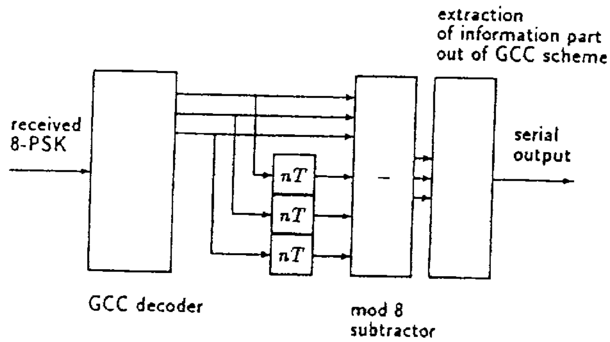

Fig. 3 - Differential decoding (8-PSK).

The decoding of the proposed coded modulation scheme is indeed of very low complexity. In multistage soft decision decoding the first code can be decoded by comparing the received (analogue) word with both codewords by means of the corresponding Euclidean distances. The second code can be decoded by the so-called Wagner decoding (which can be regarded as a special case of Chase's algorithm). This means inverting the least reliable bit of the parity check code if the parity is odd. The described procedure is a maximum-likelihood (ML) decoder for each stage. With little more computational effort, an overall maximum-likelihood decoder can be realized by means of the 4-state trellis according to Fig. 4 (see, e.g., [12, 13]). The numbers at the edges of the graph correspond to the binary numbering of the 8-PSK set in Fig. 1. The trellis contains parallel transitions due to the uncoded bits of $\mathcal{A}^{(3)}$. The states in the middle part of the trellis stand for the following properties:

state 1 . . . $A^{(1)} = (0, 1, 0, 1, \ldots, 1)$, parity of $A^{(2)}$ even

state 2 . . . $A^{(1)} = (0, 1, 0, 1, \ldots, 1)$, parity of $A^{(2)}$ odd

state 3 . . . $A^{(1)} = (1, 0, 1, 0, \ldots, 0)$, parity of $A^{(2)}$ even

state 4 . . . $A^{(1)} = (1, 0, 1, 0, \ldots, 0)$, parity of $A^{(2)}$ odd

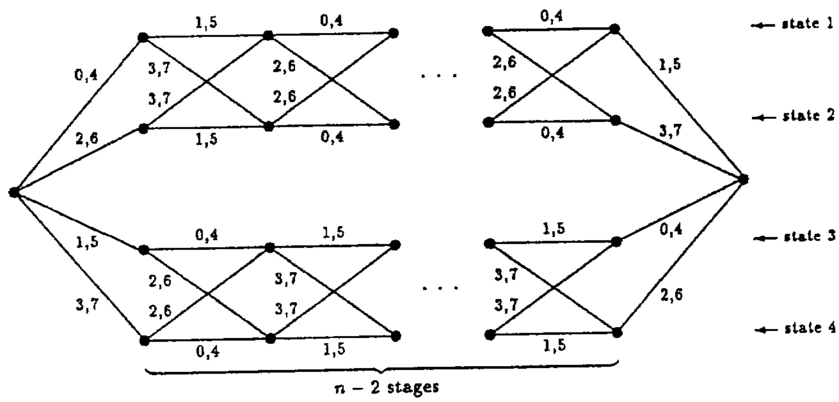

Fig. 4 - 4-state trellis of the proposed GCC scheme with differential coding.

However, the difference in error performance between multistage and overall maximum-likelihood decoding will prove to be negligible.

One problem remains to be solved. The $\pi/4$-QPSK succession has only been guaranteed inside the code block but not necessarily during the transition to the next block. Fortunately, only 180°-transitions have to be avoided to stabilize the envelope of the signal. Such a transition is given below:

$$\left|\begin{array}{cccccccc|cccccccc} 0&1&0&1&0&1&0&1&1&0&1&0&1&0&1&0\\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&0&0&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\boxed{1}&\boxed{0}&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot \end{array}\right| \quad (4)$$

Assuming that the parity bits of the second code are not located at the boundaries of the code blocks, one can use a ternary alphabet, combining the first components of the codes (2) and (3) and avoiding the 180°-transition between the last symbol of the preceding block and the first symbol of the block under consideration. This means a slight reduction of the overall code rate. E.g., 14 ternary symbols will be needed to encode 11 quaternary symbols. This follows from the relation

$$3^{i_3} \geq 4^{i_4} \Rightarrow i_3 \cdot \log 3 \geq i_4 \cdot \log 4$$
$$\Rightarrow i_3 \geq i_4 \cdot \frac{\log 4}{\log 3} = i_4 \cdot 1.262 \quad (5)$$

where i_3 ternary symbols are used to represent i_4 quaternary symbols.

The rate is reduced from 2/3 to

$$R = \frac{1 + (n-2) + (n-1) + \log_2 3}{3 \cdot n}.$$
$$(= 0.6494 \text{ for } n = 8)$$

One might also describe the abovementioned measure against 180°-transitions as follows. Considering the last components of codeword (2) and (3) of the preceding block and the first components of the current block, there are 4^2 pairs, of which 4 pairs cause 180°-trajectories. The rate is again given by

$$R = \frac{1 + (n-3) + (n-2) + \log_2 (16-4)}{3 \cdot n} =$$
$$= \frac{1 + (n-2) + (n-1) + \log_2 3}{3 \cdot n}$$

A third possibility would be to connect the trellises of succeeding blocks and delete those branches which correspond to forbidden 180°-transitions.

When plotting the bit-error probability against the signal-to-noise ratio per bit E_b/N_o the coding gain is reduced by

$$10 \log_{10} \frac{2/3}{R}$$

which equals 0.1141 dB for $n = 8$. To compensate for the reduced rate, one has to increase the symbol rate on the channel or otherwise decrease the information data rate by 2.6 % ($n = 8$). The following table gives the percentage for different block lengths n

n	8	12	16
$\Delta B/\%$	2.6	1.7	1.3

(6)

Trajectories between phase states are shown in Fig. 5.

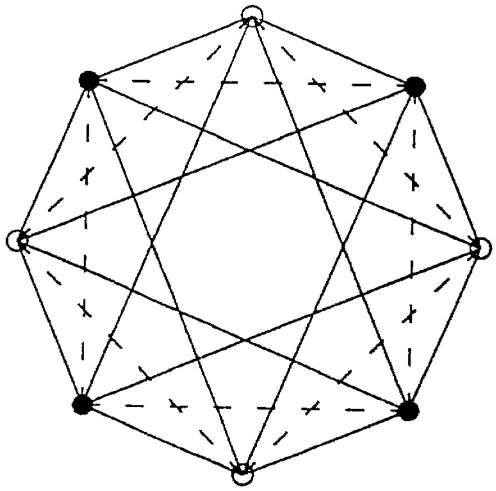

Fig. 5 - Possible trajectories between phase states for multilevel block-coded $\pi/4$-QPSK. The dashed trajectories can only occur at the block boundaries

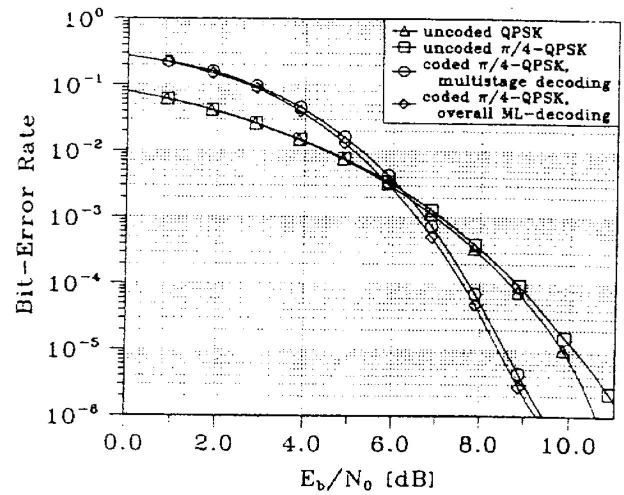

Fig. 7 - Simulated bit-error rates for the bandlimited nonlinear channel.

If a change in symbol or bit rate is impossible, one may investigate, whether a 180°-transition only between blocks is acceptable. Such a transition occurs with a probability of 1/8, so that on the average only $1/(8\,n)$ of all symbol transitions are 180°-transitions.

4. SIMULATION RESULTS

To evaluate the performance of the system, simulations were carried out for the AWGN-channel and a bandlimited, nonlinear distorting channel. A block length of $n = 8$ was chosen. The resulting bit-error rates (BER) are shown in the Figs. 6 and 7, respectively, as a function of the signal-to-noise ratio per bit E_b / N_o. In the AWGN case, conventional QPSK and uncoded $\pi/4$-QPSK with coherent detection yield of course identical results. With overall maximum-likelihood decoding, the GCC scheme provides a coding gain of about 1.7 dB at a BER of 10^{-6}. The loss in performance due to suboptimal multistage decoding is very small ($\approx$ 0.15 dB). The asymptotic coding gain is not yet achieved, but we must bear in mind that the coding scheme additionally provides the property of rotational invariance, which is especially important for the application to fading channels. If no differential encoding is employed, the BER of the coded scheme is reduced by a factor of approximately 1/2.

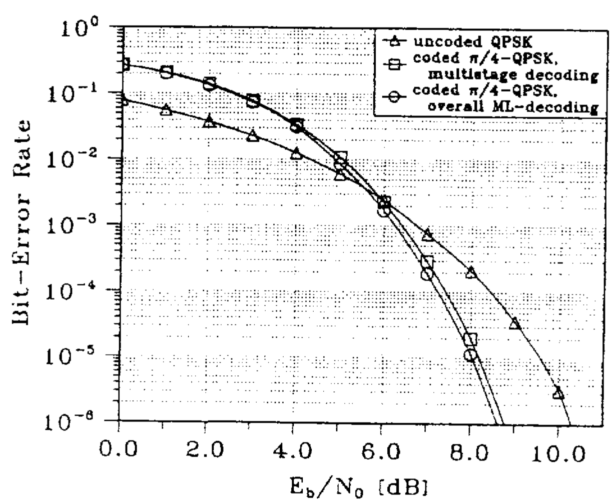

Fig. 6 - Simulated bit-error rates for the AWGN channel.

For the nonlinear channel, the modulated signal was first bandlimited by a square-root raised-cosine transmit-filter with a roll-off factor of 0.5 and then distorted by a travelling wave tube amplifier (TWTA) described as a frequency-independent memoryless bandpass nonlinearity by means of the two-parameter formulas given in [16]. The TWTA was operated at its saturation point, i.e., with no backoff, and therefore gives rise to strong nonlinear distortions in amplitude (AM/AM conversion) and phase (AM/PM conversion). This simulation model thus represents a worst-case scenario of a nonlinear channel. At the receiver, a filter matched to the transmit-filter was used in front of the GCC decoder. Since transmit and receive filter are chosen according to the Nyquist criterion, they do not introduce any linear intersymbol interference. Due to its higher susceptibility to nonlinear intersymbol interference caused by the TWTA, the $\pi/4$-QPSK signal suffers a little bit more from the distortions than a conventional QPSK signal, as can be seen in Fig. 7 at low error rates. The coding gain of the GCC scheme is preserved. Note that interference from adjacent channels was not taken into account in the simulations.

Fig. 8 presents the estimated power spectral density (PSD) of the bandlimited, nonlinearly distorted signal after the TWTA for different modulation schemes as a function of the normalized frequency $f \cdot T_{Sym}$, where T_{Sym} is the symbol duration. For comparison, the PSD of a signal without nonlinear distortions is given, too.

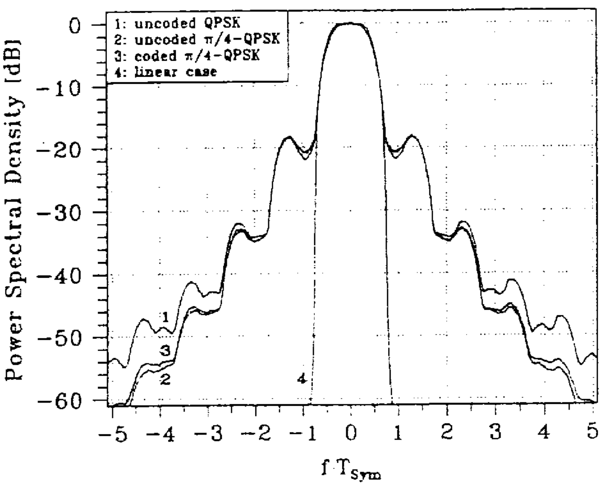

Fig. 8 - Power spectral densities of the transmitted signal for different modulation schemes (linear and nonlinear case).

The curves were calculated by averaging modified periodograms with the application of a Hanning window in the time domain. As can be seen, the TWTA causes severe spectral spreading in regenerating the spectrum sidelobes of the transmitted signal. The difference between QPSK, uncoded $\pi/4$-QPSK and block-coded $\pi/4$-QPSK is rather small. In particular the permitted 180°-transitions at the block boundaries only slightly affect the spectrum for the chosen block length of $n = 8$. Hence, it is not necessary to avoid them by the additional constraints derived in the previous chapter. However, for the considered nonlinearity the $\pi/4$-QPSK modulation scheme is not very advantageous in terms of spectral efficiency compared to conventional QPSK.

5. CONCLUSIONS

A simple multilevel block-coded modulation scheme has been proposed, which meets the constraints for the symbol transitions according to $\pi/4$-QPSK inside the blocks to reduce envelope fluctuations. Methods to avoid 180°-transitions at the block boundaries were discussed, too. The scheme is rotationally invariant and provides a coding gain of about 2 dB with low complexity. These properties make it suitable for satellite communication and mobile radio systems, where transmitters with nonlinear high-power amplifiers are used, which must be operated near saturation for reasons of power efficiency.

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