Measures to counteract the influence of phase instabilities on coded-modulation systems

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Abstract —

An overview of countermeasures agains phase instabilities affecting coded-modulation systems is given. It comprises two different proposals. One method tries to preserve the stability properties of the uncoded modulation by periodically inserting samples that are restricted to the uncoded modulation alphabet (time-variant coded modulation). A possibility of introducing a frame synchronization without any loss in data rate is pointed out. As second proposal, rotationally invariant coding is described. All coding schemes are based on so-called Multilevel Codes, where the component codes are mostly block codes. The rotationally invariant modulation codes achieve asymptotic coding gains of up to 6 dB.

1 Introduction

Since Ungerböck's trellis-coded 8-PSK had been studied for real satellite channels, it became obvious that the expected coding gain is reduced by 'cycle slips' of the carrier loop leading to long error bursts. This is due to the fact that, of course, the retention range of the carrier loop for 8-PSK is only half as wide as that for 4-PSK. Most of the convolutional codes used, are 180^o-invariant with a wide random walk zone between adjacent retention ranges of size $(-\pi/8, \pi/8]$. If a phase jitter is stronger than $\pm\pi/8$, the phase loop falls out of track, causing a long error burst.

The probability of such a failure of the carrier loop is reduced if the retention range could be increased. This is possible at least for a part of the transmitted symbols if they are taken from a subset of the modulation alphabet, e.g. 4-PSK in the case of coded 8-PSK. This means a time-variant modulation code that periodically changes the modulation alphabet.

Another possible alternative is to choose rotationally invariant modulation codes (45^o-invariant for 8-PSK). Random walks are avoided. A phase instability only causes the carrier loop to pass over to a new stable working point. Of course, errors occur during a transition, but the length of such error events is reduced.

Both measures are not necessarily independent, but may be combined to reduce the probability of error bursts and shorten them if they occur.

Time-variant as well as rotationally invariant coded modulation will be described in this contribution. The underlying construction principle is established by multilevel codes (or Generalized Concatenated Codes, GCC, [1], [2]), where the component codes are mostly block codes. The next section is devoted to time-variant coding, whereas the remainder deals with rotational invariance. $2\pi/M$-invariant M-PSK and $\pi/2$-invariant M-QAM are treated.

2 Time-variant block-coded modulation

In time-variant coded modulation the code or a time-variant mapping periodically restricts the modulation alphabet to a subset. The idea is due to Hagenauer/Sundberg [4] and Bertelsmeier/Komp [5]. Figure 1 shows the structure consisting of a rate $(n-1)/n$ convolutional code and a time-variant mapping.

A computer search was necessary to obtain convolutional codes with desired Euclidean distances.

The appointment of suitable codes is simplified if multilevel block codes are chosen. For

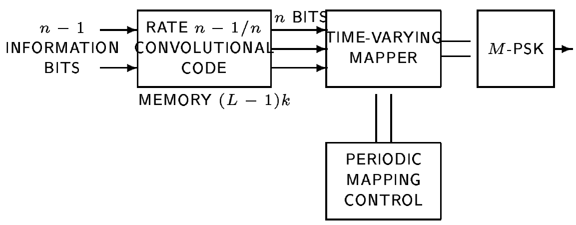

Figure 1: Time-variant convolutionally encoded modulation

a M-PSK (or even M-QAM), $log_2 M$ binary component codes of equal length n are necessary to protect the $log_2 M$ binary partitions of the modulation alphabet. These component codes may be gathered in a matrix, where the columns represent the numbering of the points of the M-PSK. For $M = 3$ this writes

$$\underline{A} = \begin{pmatrix} a^{(1)} \\ a^{(2)} \\ a^{(3)} \end{pmatrix} = \begin{pmatrix} a_1^{(1)}, a_2^{(1)}, \ldots, a_n^{(1)} \\ a_1^{(2)}, a_2^{(2)}, \ldots, a_n^{(2)} \\ a_1^{(3)}, a_2^{(3)}, \ldots, a_n^{(3)} \end{pmatrix} \begin{matrix} \leftarrow \in \mathcal{A}^{(1)} \\ \leftarrow \in \mathcal{A}^{(2)} \\ \leftarrow \in \mathcal{A}^{(3)} \end{matrix} \quad (1)$$

The corresponding set partitioning of the 8-PSK is shown in Figure 3.

The minimum quadratic Euclidean distance of two such schemes is known to be

$$d_{E_{min}} = \min_j \{ d_H^{(j)} \cdot d_E^{(j)} \} ,$$

where $d_H^{(j)}$ is the minimum Hamming distance of the j-th component code and $d_E^{(j)}$ is the minimum quadratic Euclidean distance between the corresponding $2^{(3-j)}$-PSK subsets.

Time-variant coded modulation is simply set up by fixing some components in the matrix, beginning with those in the first row. E.g. fixing components in only the first row of an 8-PSK scheme means the restriction to 4-PSK. One has to notice that the Hamming distance of a row code is decreased by the number $n_f^{(j)}$ of fixed components. If the product of Euclidean times Hamming distances corresponding to each row should be chosen uniformly, or at least decreasing with row index j one has to ensure that

$$(d_H^{(j)} - n_f^{(j)}) d_E^{(j)} \geq (d_H^{(j+1)} - n_f^{(j+1)}) d_E^{(j+1)} . \quad (2)$$

As will be seen from some typical examples of multilevel coded 8-PSK in the next section, often we have

$$d_H^{(1)} d_E^{(1)} > d_H^{(2)} d_E^{(2)} , \; d_H^{(3)} d_E^{(3)} . \quad (3)$$

This allows to fix one or two components in a frame of length n to 4-PSK, without any reduction in asymptotic coding gain.

This fixed components can also be used to introduce a frame synchronization sequence (e.g. Barker or m-sequence), not reducing the available data rate (see [6] for a detailed description). The fixed components of e.g. the first row code for coded 8-PSK are simply alternated according to the synchronization sequence. With it the 4-PSK subsets of the 8-PSK are alternating. An example with a Barker sequence of length 7 is given in Figure 2.

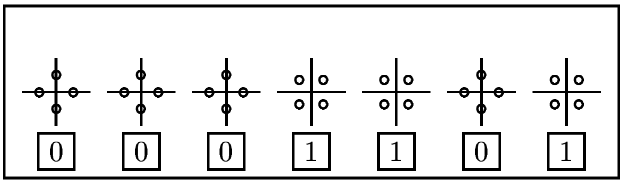

Figure 2: Barker sequence of length 7 (0,0,0,1,1,0,1), represented by the selection of 4-PSK subsets from the 8-PSK

The original binary synchronization sequence is recovered by exponentiation with four. The maximum of the absolute value of a complex crosscorrelation with the original binary sequence indicates the position of frame synchronization and the corresponding phase equals the phase offset (times 4), if it is within the retention range of $(-\pi/4, +\pi/4]$. Additionally, a symbol synchronization can be achieved by oversampling the crosscorrelation.

3 Rotationally invariant multilevel coded modulation

The second alternative to resist phase instabilities is rotational invariance. Several proposals concerning this subject can be found in literature:

- ⋆ Oerder, Meyr (6- und 8-state trellis codes)
- ⋆ Ungerböck et al. (nonlinear trellis codes)
- ⋆ Wei, Pietrobon et al. (multidimensional trellis codes)
- ⋆ Massey et al. (ring codes)
- ⋆ Lin, Kasami, et al., Henkel (multilevel block codes)

This contribution concentrates on the last principle. First of all, conditions are developed that have to be fulfilled to ensure rotational invariance. M-PSK is treated first, differences for M-QAM are pointed out afterwards.

3.1 Necessary and sufficient conditions for phase invariance with respect to multiples of $2\pi/M$

A signal space code is rotationally invariant with respect to *multiples* of $2\pi/M$ if and only if it is invariant with respect to a $2\pi/M$ rotation. Therefore we need to consider only the rotation by $2\pi/M$.

The $2\pi/M$ rotation is equivalent to the addition of $(r^{(k)}, r^{(k-1)}, \ldots, r^{(2)}, r^{(1)}) = (0, \ldots, 0, 1)$ to each $(a_i^{(k)}, a_i^{(k-1)}, \ldots, a_i^{(2)}, a_i^{(1)})$, $i = 1, \ldots, n$, which again equals the addition mod M of the all-ones vector to the base M representation (octal for 8-PSK) of $(a_i^{(k)}, a_i^{(k-1)}, \ldots, a_i^{(2)}, a_i^{(1)})$.

For illustration, the following table shows the addition of $(0,0,0,1) \equiv \pi/8$ to all possible 4-tuples of the 16-PSK.

Changes in $a_i^{(2)}$, $a_i^{(3)}$, and $a_i^{(4)}$ are framed.

We observe that the binary numbering is changed according to

$$a_i^{(1)}|_{+2\pi/M} = a_i^{(1)} + 1 \mod 2 \tag{4}$$

$$a_i^{(j)}|_{+2\pi/M} = a_i^{(j)} + \prod_{\eta=1}^{j-1} a_i^{(\eta)} \mod 2\,, \quad j = 2, \ldots, k\,. \tag{5}$$

$+(0,0,0,1),\ +\pi/8$

0	0	0	0	⟶	0	0	0	1
0	0	$\boxed{0}$	1	⟶	0	0	$\boxed{1}$	0
0	0	1	0	⟶	0	0	1	1
0	$\boxed{0}$	$\boxed{1}$	1	⟶	0	$\boxed{1}$	$\boxed{0}$	0
0	1	0	0	⟶	0	1	0	1
0	1	$\boxed{0}$	1	⟶	0	1	$\boxed{1}$	0
0	1	1	0	⟶	0	1	1	1
$\boxed{0}$	$\boxed{1}$	$\boxed{1}$	1	⟶	$\boxed{1}$	$\boxed{0}$	$\boxed{0}$	0
1	0	0	0	⟶	1	0	0	1
1	0	$\boxed{0}$	1	⟶	1	0	$\boxed{1}$	0
1	0	1	0	⟶	1	0	1	1
1	$\boxed{0}$	$\boxed{1}$	1	⟶	1	$\boxed{1}$	$\boxed{0}$	0
1	1	0	0	⟶	1	1	0	1
1	1	$\boxed{0}$	1	⟶	1	1	$\boxed{1}$	0
1	1	1	0	⟶	1	1	1	1
$\boxed{1}$	$\boxed{1}$	$\boxed{1}$	1	⟶	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	0

Together with the assumption of linearity, the necessary and sufficient conditions for phase-invariant coded modulation based on Zinoviev's scheme are:

$$\boxed{\begin{array}{c} (1,1,...,1) \in \mathcal{A}^{(1)}, \\ \prod_{m=1}^{j-1} a^{(m)} \in \mathcal{A}^{(j)}\,, \quad j = 2, \ldots, k \end{array}} \tag{6}$$

The following section shows that Reed-Muller codes as outer codes easily fulfil these conditions if they are chosen in a special *increasing order.*

3.2 Combining Reed-Muller codes to form phase-invariant PSK

An r-th order Reed-Muller code RM(r, m) of block length 2^m can be defined by a block generator matrix of the form (see e.g. [7]

$$G = \begin{pmatrix} G_0 \\ G_1 \\ \vdots \\ G_r \end{pmatrix}, \tag{7}$$

where G_0 is the all-ones vector of length 2^m. G_1 is an $m \times 2^m$-matrix, consisting of each binary m-tuple appearing once as a column. G_l ($2 \leq l \leq r$) is formed by all different products of l rows of G_1. Thus, the number of information bits follow to be

$$k = 1 + \binom{m}{1} + \ldots + \binom{m}{r} . \qquad (8)$$

The minimum Hamming distance can be shown to be $d_H = 2^{m-r}$.

We observe that $\mathrm{RM}(0,m) \subset \mathrm{RM}(1,m) \subset \mathrm{RM}(2,m) \subset \ldots \subset \mathrm{RM}(m,m)$ and in view of the conditions for phase-invariance derived above:

$$\boxed{\begin{array}{l} (1,1,\ldots,1) \in \mathrm{RM}(r,m) , \\ a_{r^{(\eta)}} \in \mathrm{RM}(r^{(\eta)},m) \Longrightarrow \\ \Longrightarrow \prod\limits_{\sum_{\eta=1}^{j-1} r^{(\eta)} \le r^{(j)}} a_{r^{(\eta)}} \in \mathrm{RM}(r^{(j)},m) . \end{array}} \qquad (9)$$

It follows that phase-invariant coded M-PSK ($M = 2^k$) is achieved if the outer codes $\mathcal{A}^{(j)}$ are chosen according to

$$\boxed{\begin{array}{c} \mathcal{A}^{(j)} = \mathrm{RM}(r^{(j)},m), \\ r^{(1)} < r^{(2)} < \ldots < r^{(k)} , \\ \sum\limits_{\eta=1}^{j-1} r^{(\eta)} \le r^{(j)} \vee r^{(j)} = m , \quad j = 2,\ldots,k , \end{array}} \qquad (10)$$

which in turn means that $\mathcal{A}^{(1)} \subset \mathcal{A}^{(2)} \subset \ldots \subset \mathcal{A}^{(k)}$, equality being excluded.

In the following table some GCC schemes with RM codes for coded 8-PSK, fulfilling these conditions, are given:

(j)	(n,k,d_H)	(r,m)	d_E	$d_H d_E$	G	R
1	(8,1,8)	(0,3)	0.586	4.688		
2	(8,7,2)	(2,3)	2	4	3	2/3
3	(8,8,1)	(3,3)	4	4		
1	(16,1,16)	(0,4)	0.586	9.376		
2	(16,11,4)	(2,4)	2	8	6	9/16
3	(16,15,2)	(3,4)	4	8		
1	(32,6,16)	(1,5)	0.586	9.376		
2	(32,26,4)	(3,5)	2	8	6	21/32
3	(32,31,2)	(4,5)	4	8		
1	(64,22,16)	(2,6)	0.586	9.376		
2	(64,57,4)	(4,6)	2	8	6	0.74
3	(64,63,2)	(5,6)	4	8		

($G := d_{E_{min}}/2\,/\mathrm{dB} = 10\log\min_j\{(d_H^{(j)} \cdot d_E^{(j)})/2\}$, where 2 is the quadratic Euclidean distance of the uncoded 4-PSK, $R = \Sigma k^{(j)}/\Sigma n^{(j)}$: coderate)

The 3-dB gain of the first example corresponds to what is achieved with 4-state trellis codes, and the other three examples are comparable to 128-state TCM.

Until now, only the rotational invariance of the code has been ensured, not yet of the information itself. A special differential en- and decoding is necessary. This results in supplementary conditions for the component codes.

3.3 Differential coding for phase-invariant GCC schemes

A possible differential en- and decoding scheme for 8-PSK is given in Figure 4 and 5, respectively. It consists of a differential encoding mod 8 over a modulation interval of block length n after the GCC encoder. The differential decoder itself has to be positioned after the GCC decoding, otherwise leading to a 3-dB loss. Thus, the GCC schemes additionally must be invariant against differential encoding mod 8.

Regarding the mod-8 addition of two codewords $A_1 \in \mathcal{A}$ and $A_2 \in \mathcal{A}$, $A_1 \neq A_2$, as performed by the differential encoder, we obtain

$$4\cdot(A_1^{(3)}+A_2^{(3)}) + 2\cdot(A_1^{(2)}+A_2^{(2)}) + (A_1^{(1)}+A_2^{(1)}) , \qquad (11)$$

where $A_k^{(j)}$ denotes the j-th binary component of the octal number. First we realize that, of course, $(A_1^{(1)} + A_2^{(1)}) \in \mathcal{A}^{(1)}$ mod 2 (linearity). A carry occurs at positions, where both $A_1^{(1)}$ and $A_2^{(1)}$ have ones or, equivalently, where the componentwise product $A_1^{(1)} \cdot A_2^{(1)}$ yields ones. Thus, products of codewords out of $\mathcal{A}^{(1)}$ have to be in $\mathcal{A}^{(2)}$.
Analysing the third binary component, we observe that similarly, products of codewords out of $\mathcal{A}^{(2)}$ have to be in $\mathcal{A}^{(3)}$ as well.

In the general case of M-PSK we obtain

$$\boxed{\begin{array}{c} A_1^{(j)} \in \mathcal{A}^{(j)}\,,\; A_2^{(j)} \in \mathcal{A}^{(j)} \\ \Longrightarrow A_1^{(j)} \cdot A_2^{(j)} \in \mathcal{A}^{(j+1)}\,, \\ j = 1, \ldots, k-1 \end{array}} \qquad (12)$$

In the case of RM(r, m) codes as outer codes this means:

$$\boxed{2 \cdot r^{(j)} \leq r^{(j+1)} \quad \vee \quad r^{(j+1)} = m} \qquad (13)$$

Codes that fulfil both conditions are given subsequently.

(j)	(n,k,d_H)	(r,m)	d_E	$d_H d_E$	$G^{(j)}$	R
1	(8,1,8)	(0,3)	0.59	4.69	3.7	
2	(8,7,2)	(2,3)	2	4	3	2/3
3	(8,8,1)	(3,3)	4	4	3	
1	(16,1,16)	(0,4)	0.59	9.38	6.71	7/12=
2	(16,11,4)	(2,4)	2	8	6	0.58 ∘
3	(16,16,1)	(4,4)	4	4	3	$< 2/3$
1	(16,5,8)	(1,4)	0.59	4.69	3.7	3/4=
2	(16,15,2)	(3,4)	2	4	3	0.75 •
3	(16,16,1)	(4,4)	4	4	3	$> 2/3$
1	(32,6,16)	(1,5)	0.59	9.38	6.71	
2	(32,26,4)	(3,5)	2	8	6	2/3 ∘
3	(32,32,1)	(5,5)	4	4	3	
1	(32,16,8)	(2,5)	0.59	4.69	3.7	79/96=
2	(32,31,2)	(4,5)	2	4	3	0.82 •
3	(32,32,1)	(5,5)	4	4	3	$> 2/3$

The schemes labelled with a '∘' are unequal error protection codes and those denoted with a '•' have rates greater than 2/3, enabling to provide additional outer codes (e.g. Reed-Solomon codes). This improves the coding gain by decreasing the overall coderate. The redundancy of the outer code may be chosen such that the original data rate of the uncoded transmission is retained ($R_{\text{total}} = 2/3$ for coded 8-PSK).

It should be mentioned that another differential encoder is currenly developed which frees from the second condition at least for the last component code. This means the codes of the first table (p. 4) being still valid. Especially the last code scheme with $n = 64$ is of some importance, because it offers a constant 6 dB gain and additionally a rate greater than 2/3 (enables outer RS code).

3.4 Conditions for 90^o-rotationally invariant M-QAM

In the case of 90^o-invariant QAM only conditions for the first two codes have to be fulfilled, because the numbering in the set partitions is chosen to be itself rotationally invariant for $(j) = (3), (4), \ldots$. The differential en-/decoding is performed in the same way, except that there are mod-4 operations concerning only the digits of the first two component codes $((j) = (1), (2))$.

The following table describes the effect of a phase shift by 90^o on 16-QAM symbols.

$+\pi/2$								
0	0	0	0	$\longrightarrow$	0	0	0	1
0	0	$\boxed{0}$	1	$\longrightarrow$	0	0	$\boxed{1}$	0
0	0	1	0	$\longrightarrow$	0	0	1	1
0	0	$\boxed{1}$	1	$\longrightarrow$	0	0	$\boxed{0}$	0
0	1	0	0	$\longrightarrow$	0	1	0	1
0	1	$\boxed{0}$	1	$\longrightarrow$	0	1	$\boxed{1}$	0
0	1	1	0	$\longrightarrow$	0	1	1	1
0	1	$\boxed{1}$	1	$\longrightarrow$	0	1	$\boxed{0}$	0
1	0	0	0	$\longrightarrow$	1	0	0	1
1	0	$\boxed{0}$	1	$\longrightarrow$	1	0	$\boxed{1}$	0
1	0	1	0	$\longrightarrow$	1	0	1	1
1	0	$\boxed{1}$	1	$\longrightarrow$	1	0	$\boxed{0}$	0
1	1	0	0	$\longrightarrow$	1	1	0	1
1	1	$\boxed{0}$	1	$\longrightarrow$	1	1	$\boxed{1}$	0
1	1	1	0	$\longrightarrow$	1	1	1	1
1	1	$\boxed{1}$	1	$\longrightarrow$	1	1	$\boxed{0}$	0

For reasons of our particular numbering of the QAM points, only the components of $a^{(1)}$ and $a^{(2)}$ are changed according to

$$\begin{array}{lcl} a_s^{(1)} & = & (1, ..., 1) + a^{(1)} \\ a_s^{(2)} & = & a^{(1)} + a^{(2)} \\ a_s^{(j)} & = & a^{(j)} \quad j = 3, \ldots, i\,. \end{array} \qquad (14)$$

Presuming linearity, we obtain the following necessary and sufficient conditions for phase invariance with respect to multiples of $\pi/2$:

$$\boxed{\begin{array}{c} (1, 1, ..., 1) \;\in\; \mathcal{A}^{(1)} \\ \mathcal{A}^{(1)} \;\subset\; \mathcal{A}^{(2)} \end{array}} \qquad (15)$$

The supplementary condition caused by the special differential encoding is also similar to the one for PSK:

$$\boxed{\begin{array}{l} A_1^{(1)} \in \mathcal{A}^{(1)}\,,\ A_2^{(1)} \in \mathcal{A}^{(1)} \\ \Longrightarrow A_1^{(1)} \cdot A_2^{(1)} \in \mathcal{A}^{(2)} \end{array}} \tag{16}$$

It is again caused by the carry in the mod-4 addition

$$2 \cdot (A_1^{(2)} + A_2^{(2)}) + (A_1^{(1)} + A_2^{(1)})\,, \tag{17}$$

where again $(A_k^{(2)}, A_k^{(1)})$ denote the binary components of the mod-4 numbers.

In the case of RM codes the two conditions can be expressed as

$$\begin{array}{ll} \mathbf{1.)} & \text{'Phase invariance' of } \mathcal{A}: \\ & r^{(2)} \geq r^{(1)} \vee r^{(2)} = m \\ \mathbf{2.)} & \text{'Differential invariance'}: \\ & r^{(2)} \geq 2 \cdot r^{(1)} \vee r^{(2)} = m \end{array} \tag{18}$$

Just as for PSK, the second condition can be omitted if another differential encoding is used (see Figure 6). However, interesting schemes (in the following table) automatically meet the second condition. (For further details see [8].)

(j)	(n,k,d_H)	(r,m)	d_E	$d_H d_E$	G	R
1	(8,4,4)	(1,3)	1	4	3	
2	(8,7,2)	(2,3)	2	4	3	0.84
3	(8,8,1)	(3,3)	4	4	3	> 3/4
4	(8,8,1)	(3,3)	8	8	(6)	
1	(16,5,8)	(1,4)	1	8	6	
2	(16,11,4)	(2,4)	2	8	6	0.73
3	(16,15,2)	(3,4)	4	8	6	< 3/4
4	(16,16,1)	(4,4)	8	8	6	
1	(32,16,8)	(1,5)	1	8	6	
2	(32,26,4)	(3,5)	2	8	6	0.82
3	(32,31,2)	(4,5)	4	8	6	> 3/4
4	(32,32,1)	(5,5)	8	8	6	

3.5 Multilevel convolutional codes

90^o-invariant QAM can also be achieved by combining suitable convolutional codes, whereas in the block-coded case, the all-ones sequence has to be a valid codesequence of the first component code and codesequences of the first code have to be valid codesequences of the second code, too. It can be shown that this implies that all generators of code one and two must have odd weight. Additionally, a special construction ensures that the second condition is also fulfilled.

To elude additional conditions due to the differential coding, the special encoder of Figure 6 has to be applied. Some results for 90^o-invariant coded QAM are given if the following table.

Code	Generators	R	L	d_f	G
1	(4,7,7)	$\frac{1}{3}$	3	6	
2	(10,13,15)	$\frac{2}{3}$	2	3	4.7
or 2	(16,13,15)	$\frac{2}{3}$	2	3	4.7
1	(15,15,13)	$\frac{1}{3}$	4	9	
2	(51,61,73)	$\frac{2}{3}$	3	5	6.5
1	(1,2,7,7)	$\frac{1}{4}$	3	8	
2	(46,52,61,73)	$\frac{3}{4}$	2	4	6.0

In computing the asymptotic coding gain, only the first two codes have been considered. The generators are given in octal representation. d_f is the free distance, $L \cdot k$ the constraint length, where $R = k/n'$. The advantage of such convolutional multilevel schemes is to be seen in the considerably low complexity. The numbers of states for the last code, e.g., are only 4 and 8, respectively.

For reasons of page limititations, the semi-algebraic construction cannot be treated in detail, but a separate publication is currently prepared.

4 Summary

Two different construction principles of modulation codes to combat with phase intabilities have been described. The first was to periodically alternate the modulation alphabet, called time-variant coded modulation. It reduces the probability of error bursts caused by cycle slips of the carrier phase. This method additionally offers the possibility to introduce frame synchronization sequences without any rate reduction. The second measure was to choose rotationally invariant modulation codes. Construction rules have been stated for multilevel block and convolutionally encoded M-PSK and M-QAM. The least decoding effort is achieved with multilevel convolutionally encoded QAM.

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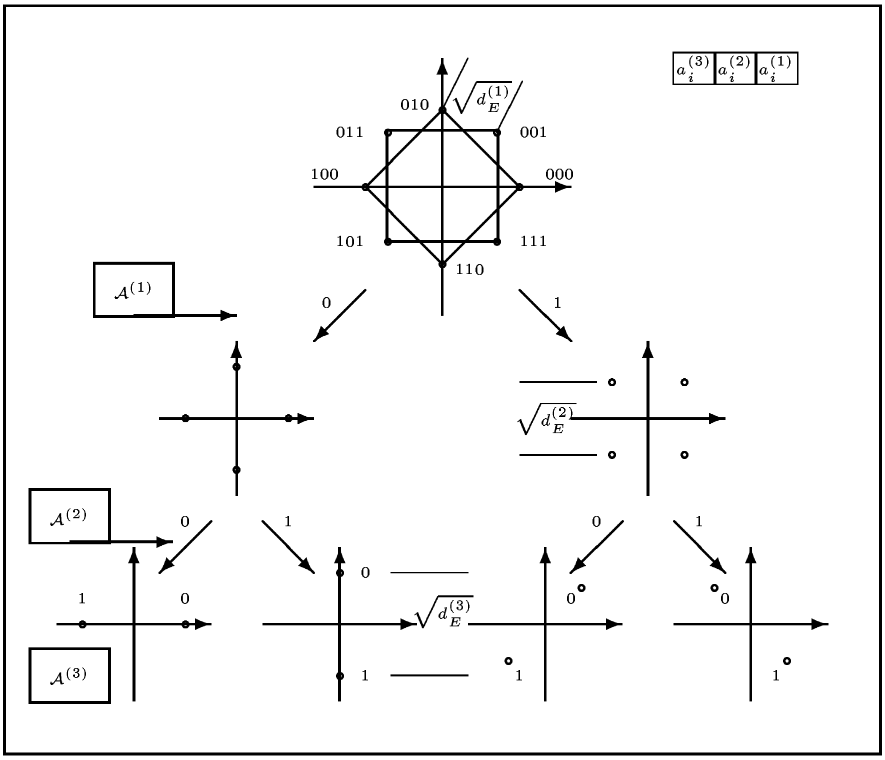

Figure 3: Set partitions of the 8-PSK

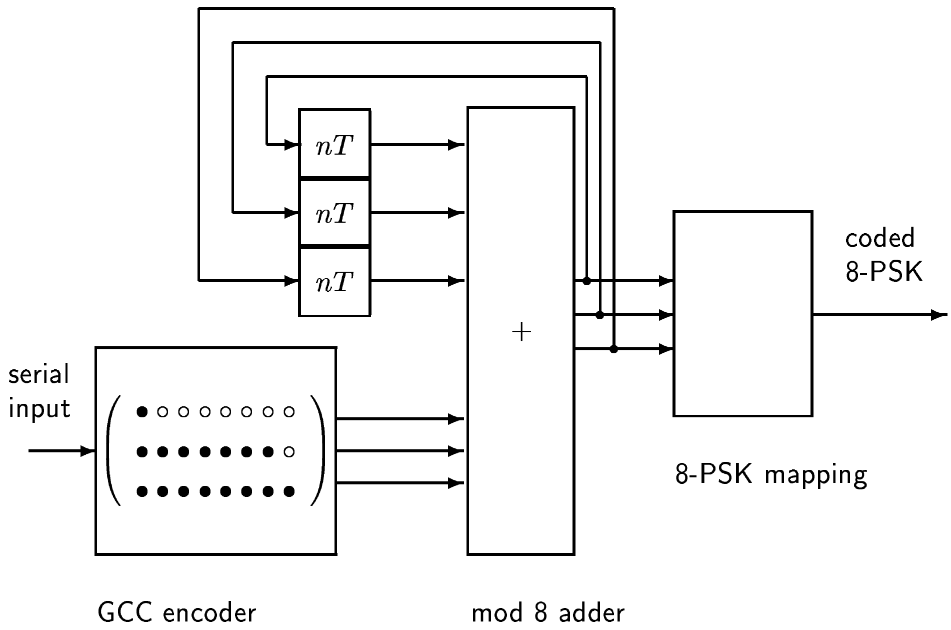

Figure 4: Differential encoding (8-PSK)

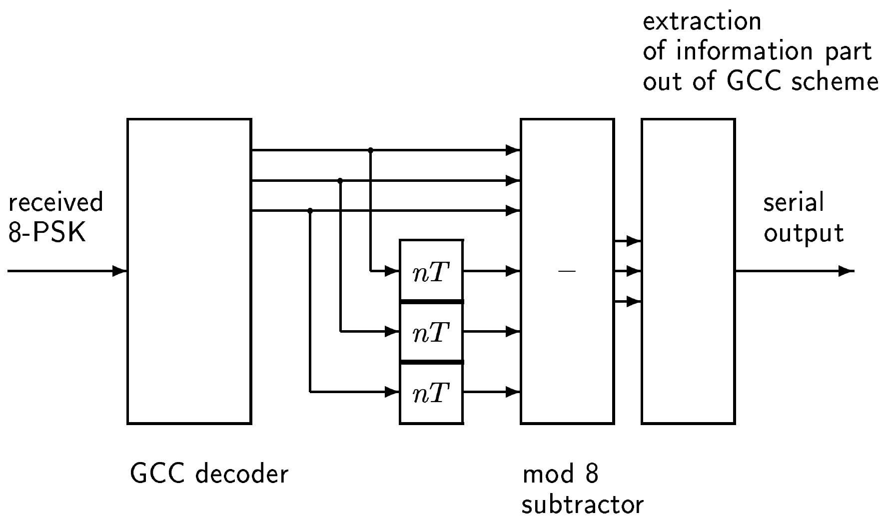

Figure 5: Differential decoding (8-PSK)

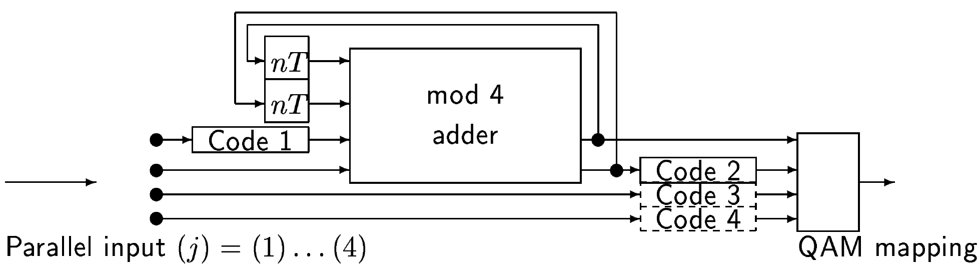

Figure 6: Modified differential encoding for 90°-phase invariant block-coded 16-QAM