Partial Transmit Sequences and Trellis Shaping

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Abstract

This paper outlines the similarities of two peak-reduction procedures called ‘Partial Transmit Sequences’ (PTS) by Mueller et al. and the one based on ‘Trellis Shaping’ by Henkel at al. From the comparison, a simplification of the PTS results that uses trellis shaping to determine the phase rotations of sub-blocks.

1 Introduction

A major drawback of multicarrier transmission is its high peak-to-average ratio (Crest factor). Due to the combination of many carriers, the time domain signal will be almost Gaussian distributed (central limit theorem). If no means of peak reduction are applied, the analog circuitry, converters, and power supplies have to be designed to handle high amplitudes even if they do not occur too often.

A manifold of procedures have been proposed so far to reduce the peaks in the time-domain signal. In here, we concentrate on two methods that have been developed independently, but due to the sub-block structure have similar properties. ‘Partial Transmit Sequences’ (PTS) by Mueller et al. [1], [2] divides the signal in DFT domain into sub-blocks and precomputes the corresponding signal vectors in time domain. Now, every sub-block and with it the corresponding time-domain signal is modified by phase rotations in order to reduce the peaks in the sum of all the modified time-domain signals. The advantage is that the procedure will operate completely in time domain. However, determining the optimum phase rotations may be quite demanding. Also one component, or at least part of it, have to be reserved to be able to determine the phase rotation at the receiver.

Trellis shaping was originally proposed by Forney in [3] as a procedure to minimize the average power, thereby leading to a Gaussian-like density distribution. The approach, however, is more general and can be used to optimize other parameters (metrics), as well. These metrics should be positive and additive. In [4] the approach is generalized to a multidimensional shaper and applied to peak-to-average ratio reduction. This was done in two ways, in time and in frequency domain. The time-domain version did not have an additive metric and thus, the trellis can just be seen as restricting a search to the trellis paths. This version will also be used to compare with PTS. Trellis shaping was designed to have an influence on the sign bits of the constellation, *i.e.*, on the MSBs in a set-partitioning scheme. For a QAM constellation, this means that trellis shaping would determine the quadrants of a signal point. However, the point has to belong to the corresponding subset, defined by the LSBs.

The second approach in [4] formulated a positive and additive metric for optimization in DFT domain. Recently, a new metric definition based on the autocorrelation function has been introduced in [5].

In the following, we will first describe PTS in some more detail and subsequently, Trellis Shaping for peak reduction. Thereafter, we compare the two schemes and describe the intermediate peak-reduction scheme, that makes use of both underlying structures. Finally, we will provide some first performance results.

2 Partial Transmit Sequences

PTS subdivides a block of length N into N_B sub-blocks of length $B = N/N_B$ as shown in figures 1 and 2. Each time-domain component vector $\mathbf{x}_i$ is computed as

$$\mathbf{x}_i = \text{IFFT}(\mathbf{X}_i)\ , \quad i = 1, \ldots, N_B \tag{1}$$

and the final time-domain signal will be computed by summing up all rotated component vectors

$$\mathbf{x} = \sum_{i=1}^{N_B} b_i \cdot \mathbf{x}_i = \sum_{i=1}^{N_B} e^{j\varphi_i} \cdot \mathbf{x}_i\ . \tag{2}$$

The phases are selected such that the peak-to-average ratio will be minimized or, at least, falls short of some limit. The complexity would grow like $N_\varphi^{N_B-1}$ for checking all possible phase choices, with N_φ being the number of possible phases and N_B the number of blocks. Subtracting one in the exponent is due to the fact that the first block can actually be left unchanged. Thus, a reasonable reduction of the investigated phase pattern is required. At the receiver, the phase rotation is detected from one reference symbol per block, which has a known phase (alternatively, differential encoding of the phases).

Part of the work was performed while W. Henkel was with Hochschule Bremen.

Note that for DMT (baseband OFDM), the conjugacy constraint between both DFT-block halves has to be taken into consideration. This leads to opposite phase shifts of the conjugate blocks in Fig. 2, *i.e.*, $\varphi_{N-i} = -\varphi_i$.

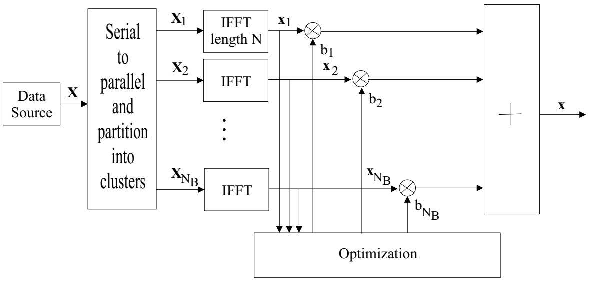

Fig. 1. Partial Transmit Sequences

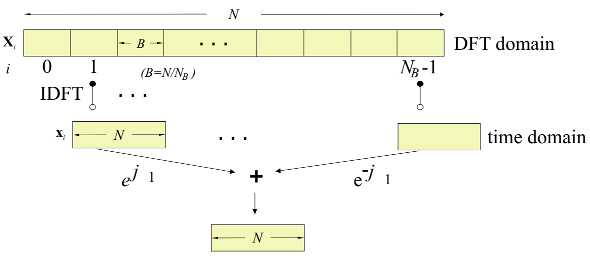

Fig. 2. The conjugate PTS phase shifts for DMT

3 Trellis shaping

A multidimensional Trellis Shaper according to [4] is depicted in Fig. 3 for the case of three bits per symbol. The transmitter-sided Viterbi-algorithm is used to block-wise modify the MSBs in a set-partitioning scheme of, *e.g.*, a QAM. This choice corresponds to selecting the point of a rotated and translated 2-PSK. Since the code sequence selected by the Viterbi algorithm is added to the MSBs, a syndrome former $\mathbf{H}^T$ is required at the receiver to eliminate the influence of the code sequence. The information has then additionally to be fed through an inverse syndrome former $\mathbf{H}^{T^{-1}}$ at the transmitter to ensure a one-to-one correspondence between input and output. The inverse syndrome former ensures orthogonality to the code sequences. To determine the metric updates inside the Viterbi algorithm, of course, not only the MSBs but also the LSBs have to be known. Usually, a Viterbi algorithm requires a positive and additive metric. If this cannot be defined, the trellis can, at least, be used to define the possible choices in a regular way. For our application, however, for intermediate path selections, running DFTs will have to be computed. For an underlying 4-state shaping code, this would mean 8 DFTs in complexity. Furthermore, no precomputation for sub-blocks would be possible using transforms of lower complexity, since the additions are not just phase

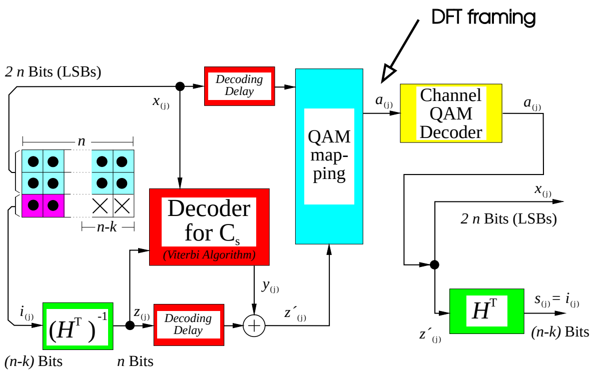

Fig. 3. Multidimensional Trellis Shaping

rotations, but choices of two alternative points, not located symmetrically to the origin.
We will see that the block-wise operation of the multidimensional shaper has some similarities to PTS.

We describe the steps again in a more formal way. The information $\mathbf{i}$ (the MSBs) is preprocessed by the inverse syndrome former

$$\mathbf{z} = \mathbf{i} \cdot (\mathbf{H}^T)^{-1} \, , \tag{3}$$

which is the counterpart of the syndrome former at the receiver eliminating the shaping code. This can be seen from

$$\mathbf{z}' \cdot \mathbf{H}^T = (\mathbf{z} \oplus \mathbf{y}) \cdot \mathbf{H}^T = (\mathbf{z} \cdot \mathbf{H}^T) \oplus \underbrace{(\mathbf{y} \cdot \mathbf{H}^T)}_{=0} = \mathbf{z} \cdot \mathbf{H}^T = \mathbf{i} \, , \tag{4}$$

where $\mathbf{y}$ is the code sequence selected by the Viterbi algorithm. $\mathbf{x}$ denotes the LSBs, also required to determine the optimized output $\mathbf{a}$. The shaping code parameters are k and n, *i.e.*, the shaping code rate is $R = k/n$. Here, n corresponds to N_B which we used before when describing PTS.

The standard convolutional codes tabulated as maximum free distance codes are suited for peak-reduction trellis shaping, as well, since they are constructed such that the Hamming distance at branches emerging from and merging into a common state is maximum, *i.e.*, they are inverse pattern. This allows for maximum phase changes in a block.

In the following section we will now propose a solution that is intermediate between PTS (Partial Transmit Sequences) and TS (Trellis Shaping) and thus, outlines the relations between both approaches.

4 The PTS/TS approach

TS was confined to just modify the MSB of the last partition. Thus, it could only select two possible signal points of a constellation. Otherwise it has a block structure like PTS. If we would use the shaping code just to address the phase changes, we would reduce the choices of the phase searches in the PTS scheme in a regular way. The syndrome former and inverse

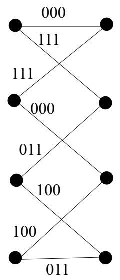

Fig. 4. Trellis segment of the 577 code

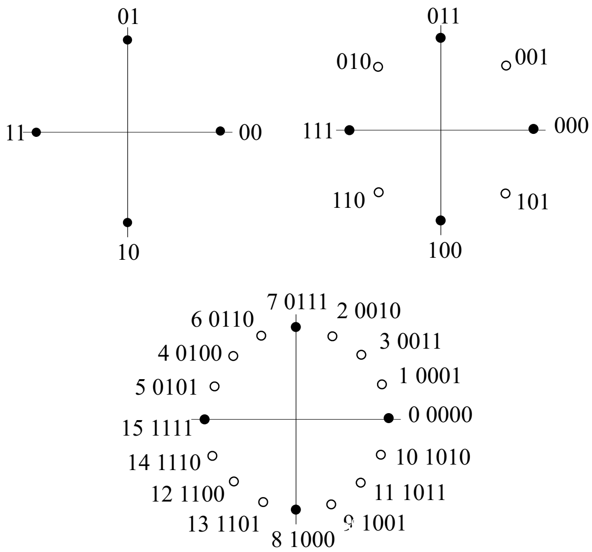

Fig. 5. Encoding phases for mixed PTS/TS

syndrome former would not be required any more. Let us assume the lowest-complexity convolutional codes, which are the ones with 4 states, the 57, 577, 5777, ... codes. They have repeated bits, which means that in each trellis segment, there are actually only four different pattern available. The trellis segment of the 577 code is shown in Fig. 4. We map the bits in the trellis onto phases according to Fig. 5. Note that the filled bullets would be the ones being chosen for these 4-state codes. The rule for the mapping between trellis bits and phases is that paths that emerge from and merge into a common state should lead to biggest phase difference. We thus map the pattern with biggest Hamming distance to the maximum phase differences.

The advantage compared to the original trellis-shaping approach is that the IFFTs can be computed first and then the phase changes are determined by the trellis without needing further transforms. Mueller reported once at a conference presentation that he had also optimized the IFFT for this special case where only a small segment is unequal to zero.

Comparing complexities, it is still not the lowest-complexity procedure. It requires N_B IFFTs and $N_B/2\cdot 2^m$ Add-Compare-Select operations where m is the memory of the shaping code and 2^m is the number of states. The additions will be over time-domain vectors of length N. Oversampling to incorporate filter functions can be easily realized, but will correspondingly increase the complexity. As far as complexity is concerned, Tellado's tone reservation [6] or its oversampled version [7] may be preferred. Nevertheless, the PTS/TS procedure may still be considered as an alternative.

5 Simulation results

For our investigations, we selected the carrier numbers of ADSL, *i.e.*, $N = 512$. All carriers except the one at DC and $N/2$ were active.

We have studied block lengths of 2,3,4,8, and 16, making use of the simplest convolutional codes of type $n_5 \times 5_8$ and $(n - n_5) \times 7_8$. Without having a table of the best convolutional codes at hand, especially they do usually not contain rate-1/16 codes, one can simply investigate the state diagram of the $(5,7)_8$ code, which is shown in Fig. 6. There one can easily see that the two paths that one would study for minimum distance are 11/01/11 and 11/10/10/11. Thus, there are especially the alternatives 01 and 10/10 in the middle. For a $1/n$-rate code, one would need to find the

$$\max_{n_5} \left[\min(n - n_5, 2 \cdot n_5)\right]$$

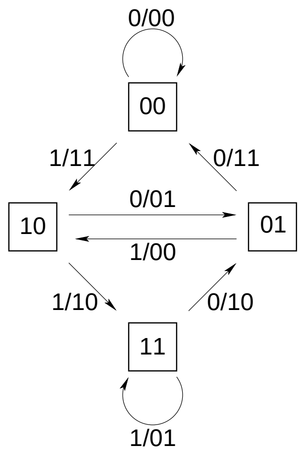

Fig. 6. State diagram of the $(5,7)_8$ code

The generators (and Hamming distances d_H) for the codes are

Rate	generators (octal)	d_H
$1/2$	57	5
$1/3$	577	8
$1/4$	5777 or 5577	10
$1/8$	55577777	21
$1/16$	5555577777777777	42
	or 5555557777777777	

We show the resulting histograms and the complementary cumulative distribution functions, *i.e.*, the clipping probability, over the PAR in figs. 7 and 8, respectively. Since we used a modified version of the original shaping program, the inverse syndrome former was still

active. This caused a slight difference in the average power of the unshaped signal. Therefore, the shaped results in Fig. 7 look similar. However, note that the unshaped curves differ, which serve as references. We clearly see the limiting effect of the procedure in Fig. 7 and a PAR reduction gain of around 5 dB at 10^{-7} for a block length of 8 and a gain of 4.5 dB for a block length of 16 follows from Fig. 8. As has been mentioned, for PTS, one symbol per sub-block has to be reserved for detecting the phase. However, some information can still be encoded in the amplitude, if required. In the case of only four alternative phases, the loss can in principal be made as small as 2 bits per sub-block.
The results are comparable to the original trellis shaping and also to Tellado's scheme. It may thus be a worthwhile alternative to Tellado's method, although it is somewhat more complex.

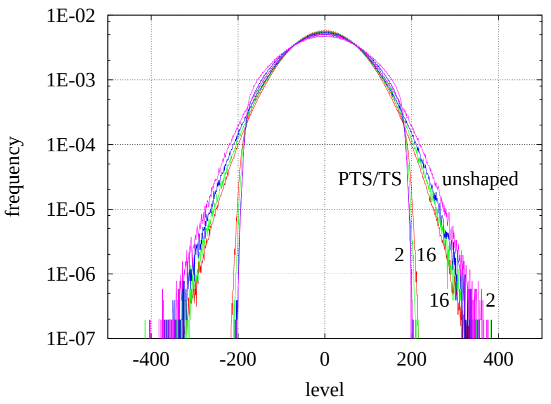

Fig. 7. Histogram of the time-domain signal applying PTS/TS

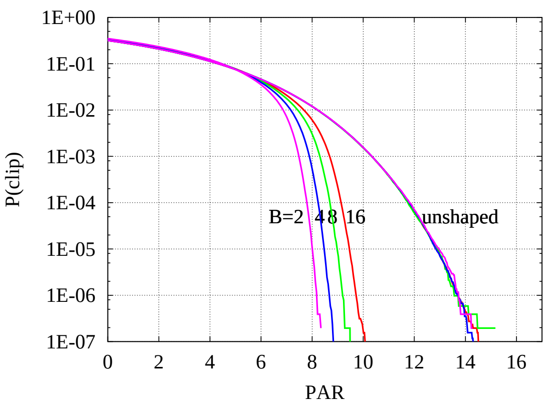

Fig. 8. The probability of clipping applying PTS/TS

6 Conclusion

From a comparison of the PAR-reduction methods 'Partial Transmit Sequences' and 'Trellis Shaping', we have developed an intermediate solution, where the shaping code determines the phase choices of PTS. Although we even restricted the phase rotations to multiples of $\pi/2$, we have obtained a substantial PAR reduction gain of around 4.5 dB reserving $6.25\,\%$ of the carriers.

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