On Code Design for Unequal Error Protection Multilevel Coding

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***Abstract*— Unequal error protection is an important feature regarding transport of multimedia data. The paper presents a novel approach for realising unequal error protection properties with multilevel codes, following the capacity design rule but adjusting the scheme to provide more unequal error protection than a multilevel coding scheme inherently provides. The flexibility of this approach is investigated regarding the freedom in the choice of unequal error protection profiles.**

I. Introduction

There are many applications in communication environments that deliver data of different error sensitivities. Especially in multimedia, there exist file formats where parts of the data are more important than others and, thus, detection errors due to additive noise or multipath propagation on the channel may have more (or less) severe effects. These different classes of importance should therefore be protected differently during transmission.

Unequal protection can be obtained in many ways and at different places in a communication scheme, e.g., it could be included into adaptive modulation or into adaptive bit and power allocation when using multicarrier modulation. This work, however, deals with unequal error protection (UEP) within coded modulation, especially multilevel codes (MLC). Coded modulation is a well-known technique which optimises the coding scheme given a certain modulation scheme [1]–[4]. Usually, the modulation alphabet is successively partitioned into smaller subsets, where each partitioning level is assigned a label. These labels are protected by separate channel codes with certain protection capabilities. The codes have to be designed carefully depending on the modulation scheme and its partitioning or labelling strategy. According to [5], the optimal way of designing the codes is to match the different code rates to the capacities of the partitioning steps. This means that, for a given signal-to-noise ratio (SNR) and given modulation scheme and partitioning, the code rates of the single codes are fixed. However, there are also other design approaches with similar results, like bit-interleaved coded modulation [6], or low-density parity-check codes optimised for a certain modulation scheme [7]. The corresponding channel codes in a multilvel coding scheme can be block codes, convolutional codes, or concatenated codes.

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Previous work on unequal error protection multilevel codes has been done in [8], [9], [10], and [11]. These publications focus on the design of special modulation schemes for achieving unequal error protection, especially with nonuniformly spaced signal constellations where symbols are grouped and the Euclidean distances within and between those groups are different. In [8], the authors additionally propose a time-division multiplexing scheme, switching between different conventional multilevel coding schemes. [9] applies non-uniform constellations together with a wavelet transform in order to perform UEP image transmission. The authors of [10] present a non-regular partitioning scheme leading to unequal error protection, and in [11], multiple block coded modulation is used together with unconventional partitioning. However, none of these publications deals with the general design of the channel codes.

In this paper, we present a way to modify the original multilevel coding approach [5] in order to obtain and control unequal error protection by defining general design rules for the coding unit. We do not limit the method to particular codes but develop rules which are applicable for any kind of codes.

The paper is structured as follows. The system model of a multilevel coding scheme is given in Section II. Modifications for obtaining unequal error protection and their results are given in Section III. Section IV finally contains a discussion about flexibility and possible improvements of the proposed scheme and conclusions are given in Section V.

II. System Model

A multilevel code consists of a modulation scheme and a coding unit. The signal constellation is successively partitioned into subsets until the subsets only contain a single signal point. The partitions are labelled at each partitioning level by symbols of an appropriate alphabet. A common approach for this partitioning strategy is Ungerböck's set partitioning, which maximises the minimum Euclidean distance between any two symbols of a subset [1], [12].

The labels of each partitioning level are components of codewords of individual codes at each level. There have been developed different design strategies for these codes. For a long time, the balanced distances rule was believed to be best

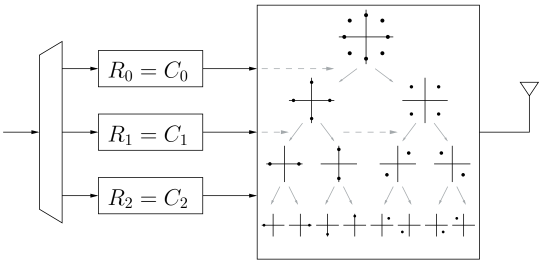

Fig. 1. Transmitter structure of an 8-PSK MLC scheme

suited, where the following holds:

$$\max \left\{ \min_{i=0,\ldots,l-1} \left\{ d_i^2 \delta_i \right\} \right\} \;, \quad \text{and therefore} \tag{1}$$

$$d_i^2 \delta_i = \text{const.} \;, \qquad i = 0, \ldots, l-1 \;, \tag{2}$$

where d_i^2 is the minimum squared Euclidean distance of the corresponding sub-constellation and δ_i is the minimum Hamming distance of the code at level i. In [5], the authors proved the capacity design rule to be optimum in terms of mutual information. According to this rule,

$$R_i = C_i \;, \tag{3}$$

which means that the code rate at each level should be equal to the capacity of the partitioning. This is connected to Shannon's theorem which states that

1) a vanishing error probability is possible for $R < C$ for block lengths tending to infinity, and
2) the error probability may never vanish for $R > C$, regardless of the block length.

As a note on this, transmitting with a code rate $R > C$ will make error-free transmission impossible, whereas a code rate $R < C$ will just reduce efficiency but maintain the possibility of error-free transmission. In the context of finite block length codes, performance is improved the farther the rate is from capacity. Figure 1 shows the structure of such a multilevel coding transmitter with 8-PSK modulation and a 3-level set partitioning.

Figure 2 shows the capacity curves for a set partitioning of an 8-PSK scheme. It contains curves for 8-PSK, QPSK, and BPSK, since the set partitioning of the 8-PSK scheme leads to these kinds of subsets. The capacities of the individual partitioning levels follow from the chain rule of mutual information, [5], and are given by

$$\begin{aligned} C_i &= I(Y; X^i | X^0 \cdots X^{i-1}) \\ &= \mathrm{E}_{x^0 \cdots x^{i-1}} \left\{ C(\mathbf{A}(x^0 \cdots x^{i-1})) \right\} - \\ &\quad\; \mathrm{E}_{x^0 \cdots x^i} \left\{ C(\mathbf{A}(x^0 \cdots x^i)) \right\} \;, \end{aligned} \tag{4}$$

where $C(\mathbf{A}(x^0 \cdots x^{i-1}))$ represents the capacity of a signal subset $\mathbf{A}(x^0 \cdots x^{i-1})$. As an example, the capacity of the first partitioning level of an 8-PSK scheme would be $C_0 = C^{\text{8-PSK}} - C^{\text{QPSK}}$.

As a decoder, a low-complexity method is given by multistage decoding (MSD), where the levels are decoded one after

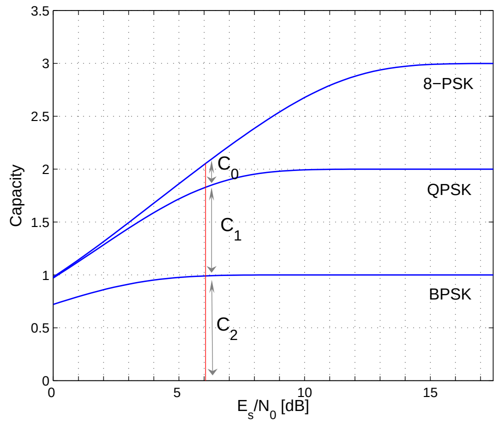

Fig. 2. Capacity curves of the partitioning levels in an 8-PSK scheme with Ungerböck's set partitioning

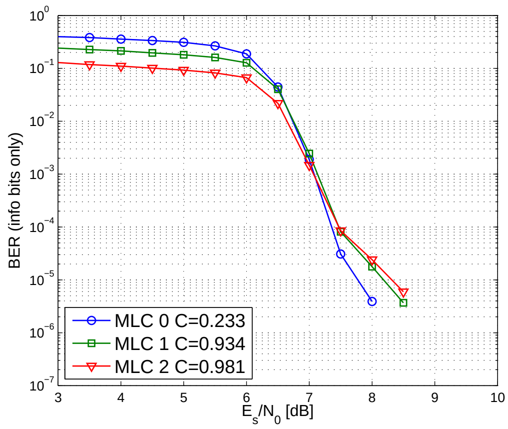

Fig. 3. Bit-error rates of the levels in an 8-PSK MLC/MSD scheme

another, taking into account decisions of previously decoded levels. Usually, for a UEP scheme, one would choose the lower partitioning levels for the most important data due to the larger Euclidean distance. However, in the case of multistage decoding, the lower levels' performance is affected by the upper levels due to error propagation. Hence, the upper levels are chosen for important data and the lower ones for less important data.

Figure 3 shows the error rates of the three levels in an 8-PSK multilevel code with multistage decoding versus the SNR. Note that the SNR is given by E_s/N_0 in dB in order to compare the levels in a fair way, taking the code rates into consideration. The codes are Turbo codes and are designed according to the capacity design rule in [5] for an operating point of $E_s/N_0 = 6$ dB. Different code rates are obtained by puncturing and pruning ([13], [14]).

III. Modification for UEP

As shown in Fig. 3, the different levels do not have significantly different performances. Thus, the question is how to modify the scheme in order to obtain more, or even a desired amount of UEP.

This work focuses on designing the coding unit rather than the modulation unit of the MLC scheme. According to the capacity design rule given in (3), the choice of the code rates is crucial to the performance of the system. The idea is now to vary the code rates in order to, on the one hand, improve the performance for the important data and, on the other hand, accept some performance loss for less important data. In spite of the lower Euclidean distance, we assign the most important data to the first partitioning level since the other levels are affected by error propagation in case of a wrong decision in the first level.

There are two intuitive ways for forcing UEP properties. The general idea is to allow for a lower code rate for the important data and increase the code rate of the less important data. The first approach is to shift the capacity curves against each other. The second idea is to choose different operating points (w.r.t. signal-to-noise ratio) for the levels.

Starting with the first approach, the capacity curves should be shifted such that the overall capacity stays the same. This allows for a fair comparison to the original non-UEP scheme. The easiest way would be to keep the capacity curve of the original constellation, only shifting the curves of the sub-constellations.

Shifting the curves has the effect that the bit-error rate curves will also be shifted. One might think that the error-rate curves are shifted by the same amount of signal-to-noise ratio as the capacity curves were shifted before. However, this is not the case. Consider the following example: Figure 2 shows the capacity curves of a multilevel code with an 8-PSK modulation scheme and, thus, three partitioning levels. For an operating point of $E_s/N_0 = 6$ dB, we have approximately the following capacities at the levels.

$$\begin{aligned} C_0 &= 0.23 \\ C_1 &= 0.84 \\ C_2 &= 0.98 \end{aligned} \tag{5}$$

Assume now, we shift the curves of the second and third level by -2 dB and -4 dB, respectively, obtaining the following capacities, see Fig. 4.

$$\begin{aligned} \tilde{C}_0 &= R_0 = 0.09 \\ \tilde{C}_1 &= R_1 = 0.94 \\ \tilde{C}_2 &= R_2 = 1 \end{aligned} \tag{6}$$

Intuitively, one might assume that the bit-error rate curves of the levels are spaced by 2 dB. Running bit-error rate computations yields, however, a quite different result, as shown in Fig. 5. The waterfall regions of the curves were expected to be located at 6 dB, 8 dB, and 10 dB, but they are in fact located at approximately 4.5 dB, 7 dB and 7 dB.

The location of the waterfall regions of the individual BER curves can be found in the following way. By designing the code rates to be equal to the capacities at a certain operating point (SNR), all levels (should) have good performance at (and above) the operating point, and a high error rate for lower SNR. By shifting the capacity curves, one obtains new

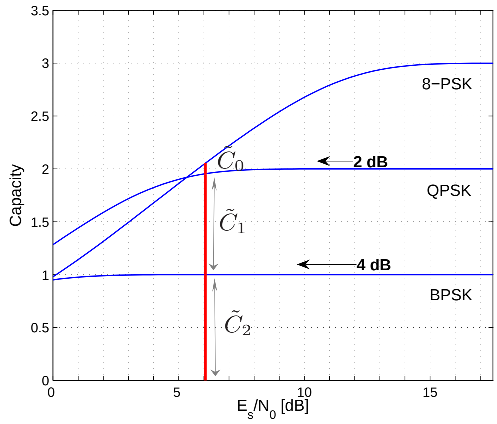

Fig. 4. Shifted capacity curves of the partitioning levels in an 8-PSK scheme with Ungerböck's set partitioning

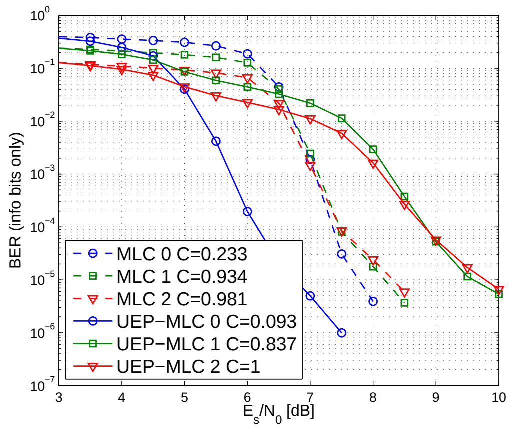

Fig. 5. Performance of an original MLC scheme and a UEP-MLC scheme with 2 dB shift in capacity curves

individual rates for the partitioning levels. Now, each level will have good performance at and above that particular SNR which yields a (true) capacity equal to the code rate on that level. Note that we assume optimal codes in this consideration. The actual location of the waterfall region depends on how close the code's performance is to the Shannon limit.

Generally, one wants at least the first level to be better protected than before, which means a reduction of its code rate. In order to keep the UEP system comparable to the original MLC system, the overall capacity shall remain equal. Assume that, for reduction of R_0, the other rates have to be increased. Comparing the new rates (6) with the original capacity curves in Fig. 2, one can find the signal-to-noise ratios where $C_i = R_i$. These signal-to-noise ratios are those where the waterfall regions will be located.

The explanation is illustrated in the unshifted capacity curve plot in Fig. 6. When partitioning level 0, which originally has the capacity given in Eq. (5), is encoded with the code rate from Eq. (6), it behaves exactly as if it would have been designed for a signal-to-noise ratio

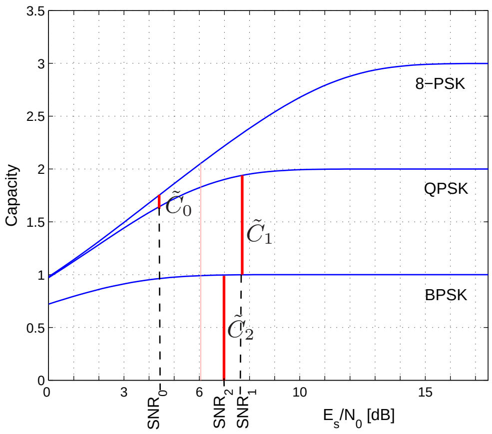

Fig. 6. Operation points leading to waterfall regions

where $C_1 = \tilde{C}_1$. This is shown in Fig. 6. The same holds for the other partitioning levels. The new locations SNR_i are now exactly the locations of the waterfall regions in Fig. 5.

IV. Discussion

Besides shifting capacity curves against each other, a second approach of achieving UEP was mentioned before where the levels' operating points are directly chosen from the capacity curves.

The first approach shown above obviously automatically leads to the second possibility. Theoretically, the waterfall regions of a desired UEP MLC scheme can therewith directly be chosen by defining appropriate code rates, as long as reasonably good channel codes are used which are near to the Shannon limit. The only constraint is that the overall rate should be kept constant in order to have comparable schemes. Generally, this method leads to a UEP system which is very easy to design and control.

Taking the capacity curves with Gray labelling into consideration, the choice of code rates can be very limited when the overall rate should be maintained. There is only a small SNR region where it is possible to trade off the code rates. Consider Fig. 2 again and imagine a desired operating point of 11 dB. The capacities of both the second and the third level are 1. The code rate of the first level can, thus, not be reduced without reducing the overall code rate.

Generally, the more levels have capacities smaller than 1, the larger the degree of freedom in the design of a UEP scheme. Nevertheless, at an operating point of 4 dB, the first level already has a very low rate and it does not make much sense to reduce it even further in favour of data throughput.

To circumvent the problem at high SNR, where $R_i = 1$, $0 < i \leq l-1$, one could still reduce the code rate of the first level. In order to maintain the throughput, an appropriate amount of information data from the last, least important level can be omitted before encoding which, in fact, leads to a rate $R_{l-1} > 1$ and allows R_0 to be reduced. In the context of scalable multimedia data, where UEP is desired, truncation of less important data in favour of more important data is a common method.

V. Conclusions

We have designed an unequal error protection multilevel coding scheme where the waterfall regions of the different protection levels can directly be chosen by trading the code rates of the partitioning levels. The constraint of a constant overall rate may reduce the flexibility of the choice of code rates. A solution is to truncate information bits from the least important data which is a common approach in the context of scalable data processing. Therewith, we have designed a flexible, easy to control UEP MLC scheme.

Acknowledgement

This work is funded by the German National Science Foundation (Deutsche Forschungsgemeinschaft, DFG).

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