A MODIFIED TRELLIS-SHAPING WITHOUT DOUBLING OF THE SYMBOL ALPHABET

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1 Introduction

Last year Forney [1] published a trellis-based approach to reduce the average transmitted signal power by achieving a Gaussian-like frequency distribution over one- or two-dimensional signal-point constellations. The proposal of the so-called *'Trellis Shaping'* has two major disadvantages:

- doubling of the signal alphabet size,
- generation of systematic errors by the Viterbi algorithm in the transmitter.

The first point seems often to be inacceptable, bearing in mind that the symbol set may already have been doubled to incorporate some sort of coded modulation. Especially, if the shaper should be applied to avoid transitions near the origin in the case of $M-PSK$ to reduce the spectral regrowth caused by nonlinearities (e.g., by a TWT), an increase from coded 8-PSK to coded and shaped 16-PSK is not feasible. This is due to severe carrier synchronization problems for higher PSK alphabets. Even in the case of PAM, for power shaping (Forney's original application) or line coding, at least, all circuitry has to be provided with an increased resolution.

A simple modification of Forney's shaper avoids the doubling of the symbol set.

Three possible applications of shaping, 'power reduction', 'line coding', and 'reduction of spectral regrowth for PSK' have already been stated. Some results are given for all of them in sections 4 to 6.

The second drawback (systematic error generation) can be avoided by a modification inside the Viterbi algorithm. This is described in Section 3, after the modified shaping has been outlined.

2 The modified shaper

The proposed modified structure is given in Fig. 1. The redundancy for shaping is now introduced sequentially (affecting only the last partition), which means a shortening of the information of a coded modulation scheme (e.g., a multilevel code). We obtain different matrix dimensions

$$(H^{-1})^T : \quad m-1 \times m\,, \qquad H^T : \quad m \times m-1$$

and a shaping convolutional code of rate $R = 1/m$.

Apart from these parameters, the structure of the shaper and the formal description (1)-(3) are the same.

$$y \cdot H^T = 0 \tag{1}$$

$$z' H^T = (z \oplus y) H^T = z H^T = s \tag{2}$$

$$z = s \cdot (H^{-1})^T \tag{3}$$

3 The modified Viterbi algorithm

The finite decoding delay of the Viterbi algorithm in the transmitter leads to systematic errors due to invalid transitions in the trellis. This can be avoided by a simple modification. One has to examine all pathregisters M[1] positions before the output position. If the contents of two such cells differ, the latest positions in the registers are searched for the best metric. This is left unchanged, whereas the others are set to bad values, so that after M steps only the path with the unchanged, good metric stays in memory. This measure ensures that all paths are always merged at the output position. An example is given in Fig. 2.

4 Results for power reduction

When using the running sum of the symbol energies as a metric, the shaper reduces the average

[1] $M = L-1$, M: memory, L: constraint length

transmitted power. The following table shows results (with $L = 4$) depending on the chosen frame length (FL) which is the inverse of the code rate of the shaping code (assuming an $R = 1/m$ code).

FL	G_1	G_2	G_F
2	3.910	1.828	0.830
3	2.753	1.481	0.705
4	2.214	1.297	0.680

G_1 is the shaping gain compared to equally distributed 8-PAM, G_2 is the gain relative to time-variant 4/8-PAM (one 4-PAM symbol and (FL-1) 8-PAM symbols), and G_F is the shaping gain according to Forney [1] which is the gain relative to an imaginary PAM with the same average rate and same Euclidean distance between its points.

The next table shows dependences on the decoding delay.

Delay	G_1	G_2	G_F
5	2.500	1.228	0.452
10	2.648	1.375	0.599
20	2.725	1.453	0.677
60	2.753	1.481	0.705
200	2.753	1.481	0.705

The frequency distribution over the 8-PAM symbols is given in Fig. 3.

5 Results for line coding

If the 'metric' is chosen to be the running digital sum (RDS) to obtain a spectral null at DC, contrasting the results of the preceding section, the spectral shaping was worse compared to an ancient line code by Carter (already published at [2]). Of course, the RDS does not fulfill the mathematical metric definitions (a metric should at least be non-negative). The sum of the squares of the momentary RDS is a much better choice. This equals the variance of the RDS, which has already been used instead of the RDS by Justesen [3]. Fig. 4 shows results with the RDS and the sum of the squares thereof as 'metric' compared to Carter's line code. The rate of the shaping code was $R = 1/3$ with a constraint length of $L = 3$. The dependance on the frame length is given in Fig. 5.

6 Results for the reduction of the spectral regrowth for PSK over nonlinear channels

In the case of nonlinearities (e.g., TWTs) a change in amplitude leads to a spectral regrowth which results in an Adjacent Channel Interference (ACI). One possible solution is to use Continuous Phase Modulation (CPM), which is in general not very spectrally efficient and has some practical disadvantages (synchronization). The only special case that is widely applied (e.g., GSM) is Minimum Shift Keying (MSK), which can be derived from an Offset QPSK, and can therefore be handled more easily.

Another possibility which has been introduced in some cellular systems is $\pi/4$-QPSK, where the phase offset of QPSK alternates between 0 and $\pi/4$. This avoids at least transitions through the origin. However, the effect on the sidelobes of the spectrum after the nonlinearity is quite small (see, e.g., [4]).

Morrison [5] proposed to use Forney's trellis shaping to reduce transitions near the origin. Unfortunately, according to Forney's paper he doubled the symbol alphabet, e.g., from 8-PSK to 16-PSK. The drawbacks are a reduction in Euclidean distance (more than for QAM) and severe carrier synchronization problems. It also seemed that his metrics were not formally derived.

We chose the 'metric' to be the power outside the mainlobe of the power density spectum and calculated it for each possible transition (elimitating the effect of a constant, which is the mean of the two chosen points of the M-PSK – except 180°-transitions. The following table shows the values for 8-PSK and square raised-cosine filtering with a roll-off of 0.35 before the nonlinearity [6].

transition	normalized metric
0°	0
45°	0.0198
90°	0.2425
135°	0.7341
180°	1

Note that $135°$- and $180°$-transitions result in nearly the same spectral regrowth.

Fig. 6 shows the spectra for different code rates, whereas Fig. 7 compares the spectra of multilevel schemes without any coding, with the code combination ((8,1,8),(8,7,2),(8,8,1)) – (R,P,U) and the one with a coset of the repetition code (R+(01010101),P,U). The given gain in the figures specifies the out-of-band power suppression. Gains can slightly be increased by increasing the number of states. E.g., the difference between $L = 3$ and $L = 9$ is about 0.5 dB ($R = 1/2$).

7 Conclusions

A modified trellis shaping without doubling of the symbol set size has been presented. It proved to be useful for power reduction, line coding and the reduction of the spectral regrowth in the case of 8-PSK on nonlinear channels (TWT).

Furthermore, a modification of the Viterbi algorithm inside the trellis shaper was proposed, which avoids systematic errors resulting from the finite decoding delay.

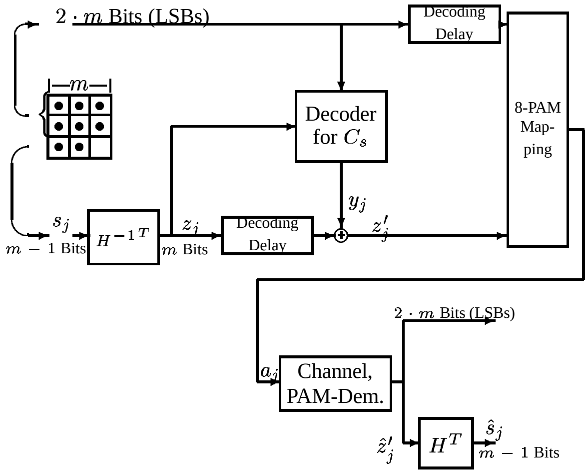

Figure 1: Modified trellis shaper

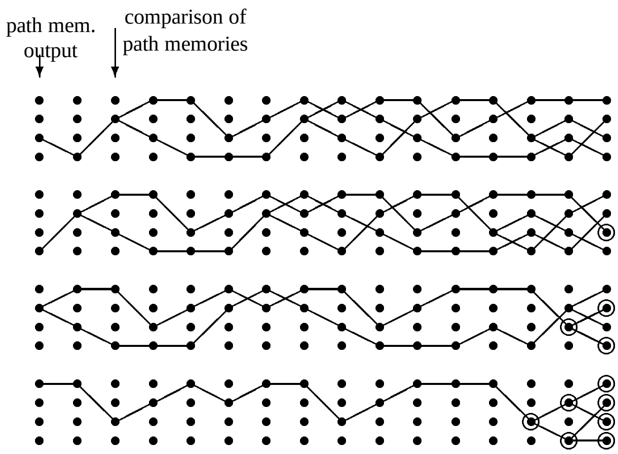

Figure 2: Modification of the Viterbi algorithm to avoid systematic error generation

References

[1] **Forney, G.D.:** Trellis Shaping, *IEEE Tr. Inf. Th.*, pp. 281-300, March 1992.

[2] **Henkel, W., Schramm, R.:** Recent Results on Trellis Shaping, *ISIT '93 - Recent Results Session*, San Antonio, USA, 17-22 Jan. 1993.

[3] **Justesen, J.:** Information Rates and Power Spectra of Digital Codes, *IEEE Tr. Inf. Th.*, pp. 457-472, May 1982.

[4] **Henkel, W., Litzenburger, M.:** A Simple Multilevel Block-Coding Scheme with $\pi/4$–QPSK Properties, to be published in *ETT*.

[5] **Morrison, I.S.:** Trellis Shaping Applied to Reducing the Envelope Fluctuations of MQAM and Bandlimited MPSK, *ICDSC-9*, pp. 143-149, Copenhagen, Denmark, 18-22 May 1992.

[6] **Saleh,˙ A.A.M.:** Frequency-Independent and Frequency-Dependent Nonlinear Models of TWT Amplifiers, *IEEE Tr. Comm.*, pp. 1715-1720, Nov. 1981.

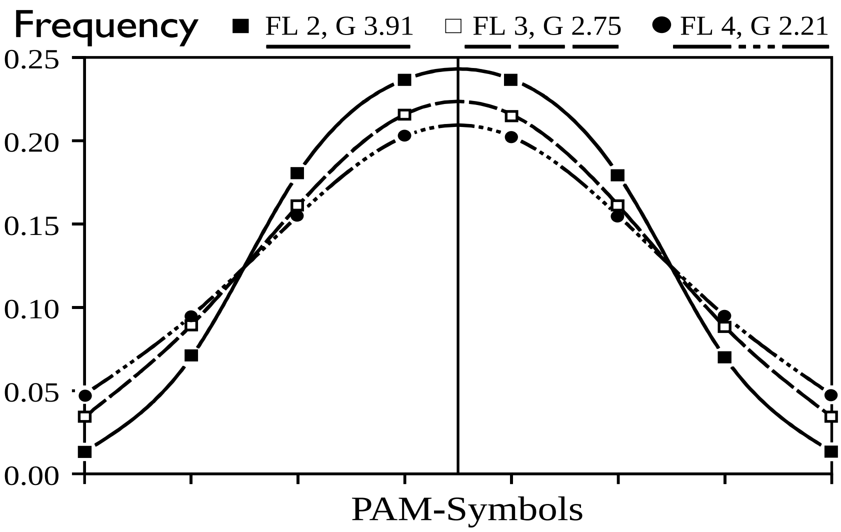

Figure 3: Frequency distribution over the 8-PAM symbols

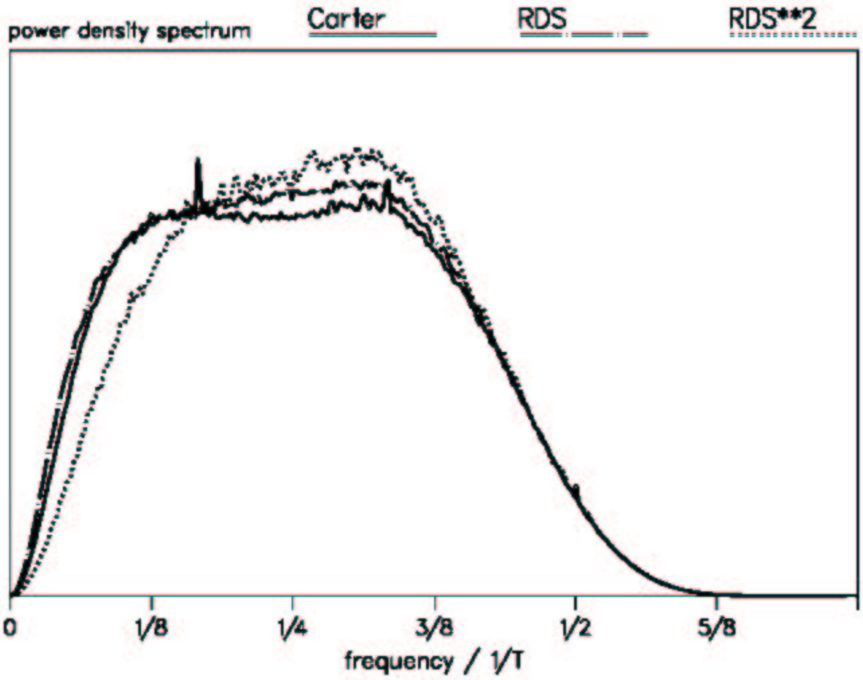

Figure 4: The modified trellis shaper with the running digital sum and its variance as metric compared to a conventional line code

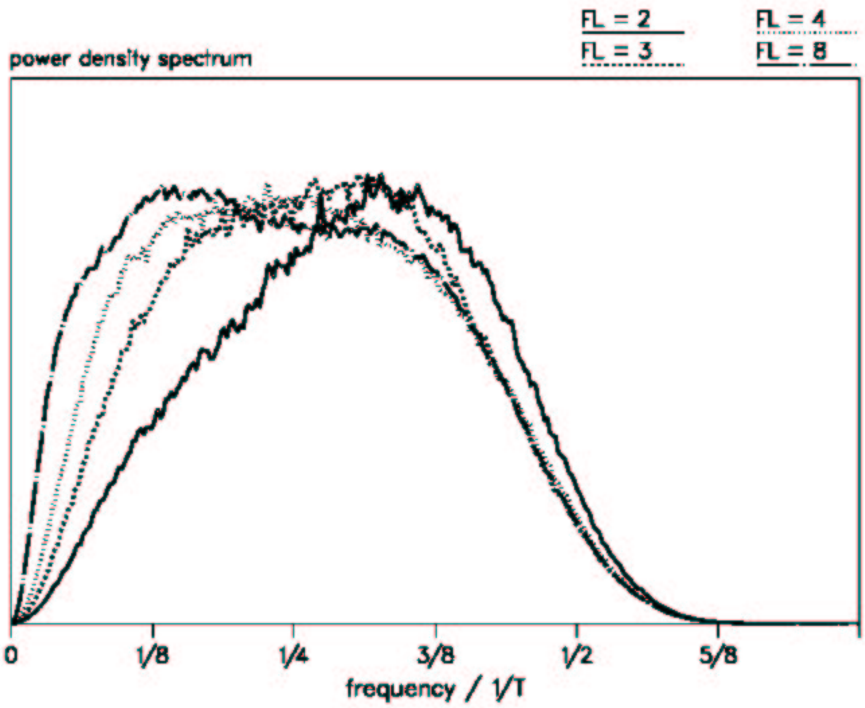

Figure 5: Line coding spectra for different frame lengths ($L = 3$, variance of the RDS as 'metric')

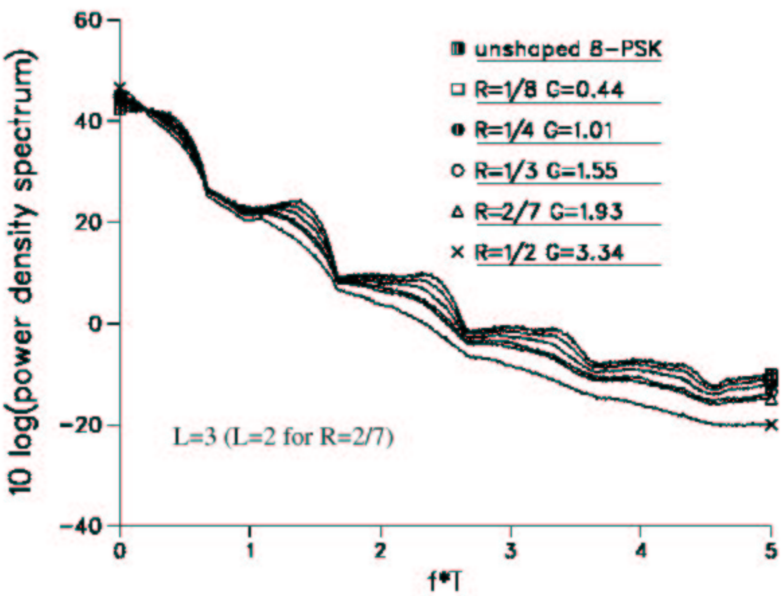

Figure 6: 8-PSK spectra after a TWT amplifier for different rates R of the shaping code

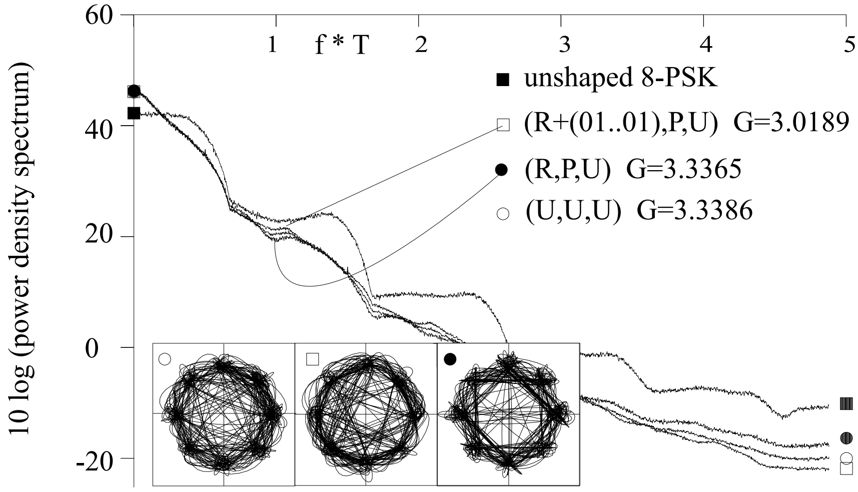

Figure 7: The influence of codes in the first two levels of a multilevel scheme on the shaping performance