Check-Irregular LDPC Codes for Unequal Error Protection Under Iterative Decoding

Lucile SASSATELLI[1,2], Werner HENKEL[1], David DECLERCQ[2]

[1]International University Bremen, Campus Ring 1 28759 Bremen, Germany

[2]ETIS ENSEA/UCP/CNRS UMR-8051, 95014 Cergy-Pontoise, France

Introduction

Context

- ◇ Unequal Error Protection (UEP): useful in the transmission of multi-media content that have heterogeneous sensibility to errors.
- ◇ LDPC codes irregularity adaptable to UEP.

Goal

- ◇ Creating flexible UEP coding scheme based on LDPC codes to process different kind of scalable data by the same system.

Approach

- ◇ Adapting the check node profile of bit-regular LDPC codes to speed up the convergence of the most protected bits.
- ◇ Flexible method consists in pruning a mother code to create several UEP subcodes decoded with the mother decoder.

Detailed Representation of Irregular LDPC Codes and Density Evolution

Detailed Representation

- ◇ To distinguish subclasses of interleavers inside one conventionnal code family [1].
- ◇ The function $\pi(b,d)$ describes the connections between the degrees b of bit nodes and the degrees d of check nodes. We can then define:

$$\lambda(b,d) = \frac{\pi(b,d)}{\sum_B \pi(b,d)} \qquad , \qquad \rho(b,d) = \frac{\pi(b,d)}{\sum_D \pi(b,d)} .$$

$\rho(b,d)$: fraction of edges connecting nodes of degree b and d among all edges of degree b.

Detailed Density Evolution with a Gaussian Approximation

- ◇ $x_{cv}^{(l)}(d)$ and $x_{vc}^{(l)}(b)$: mutual information between the input of the channel and the messages from check (bit) nodes of degree d (b) to any bit (check) node at the lth iteration.

$$x_{cv}^{(l)}(d) = 1 - J\left((d-1)J^{-1}\left(1 - \sum_{b\in B}\lambda(b,d)x_{vc}^{(l)}(b)\right)\right) ,$$

$$x_{vc}^{(l)}(b) = J\left(s + (b-1)J^{-1}\left(\sum_{d\in D}\rho(b,d)x_{cv}^{(l-1)}(d)\right)\right)$$

with $J(m) = 1 - \mathbb{E}_x(\log_2(1+e^{-x})), \quad x \sim N(m, 2m)$.

Classes of Protection

- ◇ Class of sensitivity Ck, defined by the source encoder, made of equal priority bits.
- ◇ $x_{cv}^{(l)(C_k)}$: average mutual information of messages coming out of the check nodes connected to C_k:

$$x_{cv}^{(l)(C_k)} = \sum_{b\in C_k}\lambda_b^{(C_k)}\sum_{d\in C_k}\rho^{(C_k)}(b,d)x_{cv}^{(l)}(d)$$

with $\rho^{(C_k)}(b,d) = \frac{\pi(b,d)}{\sum_{d\in C_k}\pi(b,d)}$

Cost Function

- ◇ Our UEP criterion is the local speed of convergence, represented by the difference between the mutual information of messages of the class C_k and the average mutual information over the whole graph. This difference can be lower bounded:

$$1 - J\left(\left(\sum_{d\in C_k}\rho^{(C_k)}(d)d - 1\right)J^{-1}(1 - x_{vc}^{(l)})\right) - x_{cv}^{(l-1)}$$

$$\leq x_{cv}^{(l)(C_k)} - x_{cv}^{(l-1)}$$

- ◇ The lower bound depends on the average check connection degree of the class C_k:

$$\overline{\rho}^{(C_k)} = \sum_{d=d_{min}^{(C_k)}}^{d_{max}^{(C_k)}}\rho^{(C_k)}(d)d$$

To maximize this difference, we have to minimize $\overline{\rho}^{(C_k)}$. The most protected classes will have the lowest average check degrees.

- ◇ We must care about the trade-off between UEP brought by breaking the concentration of the check node profile and the increase of the gap to the capacity of the overall code [2].

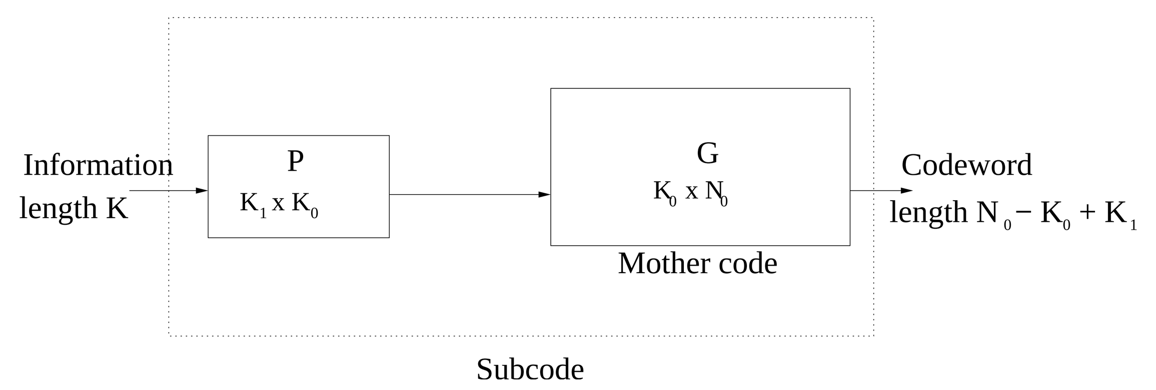
A Practical Means to Achieve UEP: Pruning a Mother Code

- ◇ Given the proportions of classes of protection:

$$\text{Columns to be pruned} = \text{argmin}\ \overline{\rho}^{(C_k)}$$

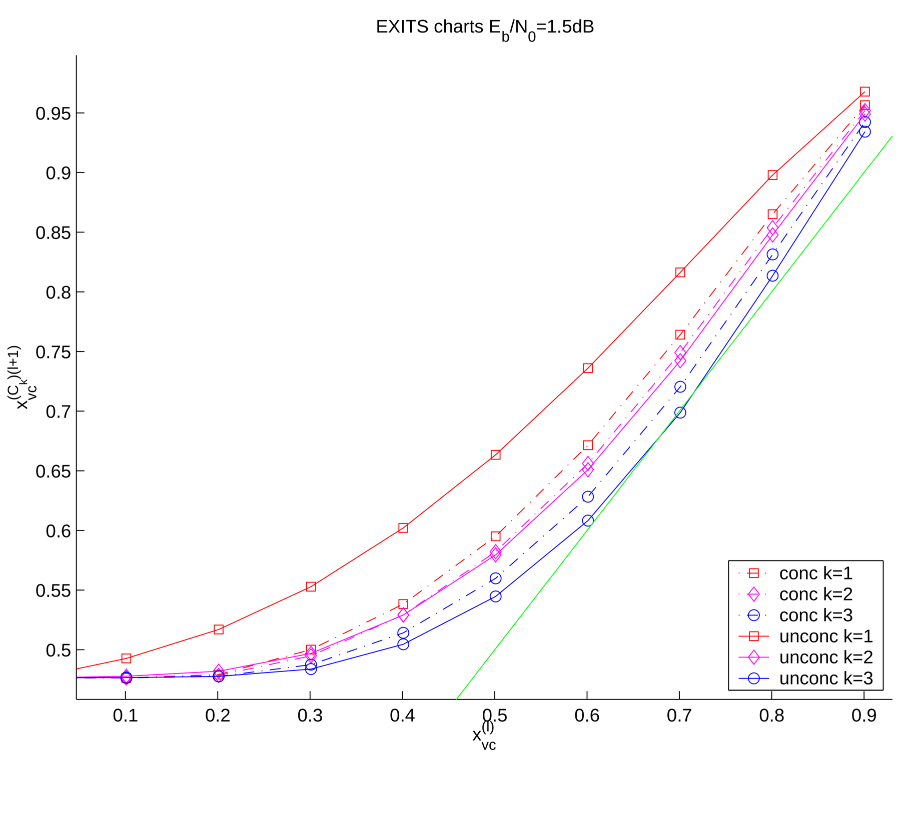
- ◇ Pruning the LDPC mother code: the pruned bits of the codeword are known by the decoder.
  → These bits disappear from the Tanner graph of the mother code, thereby reducing some check nodes degrees, and yield the UEP subcode.

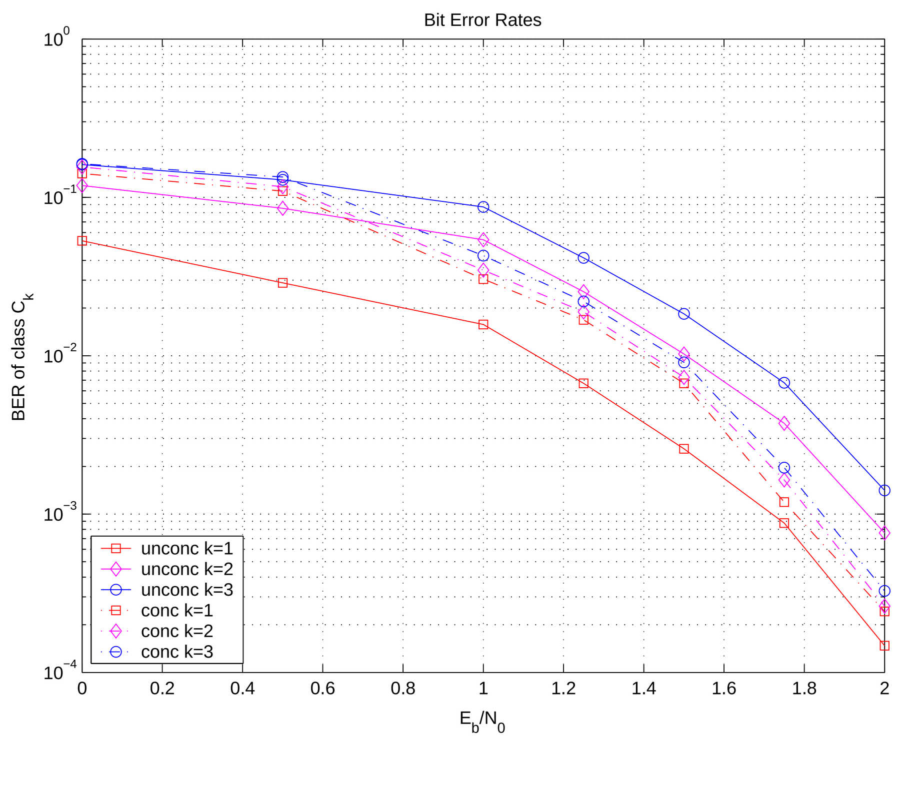
- ◇ The preprocessing matrix $\mathbf{P}$ fixes to zero $K_0 - K_1$ bits of the mother code to construct a subcode of dimension K_1, length $N_1 = N_0 - (K_0 - K_1)$ and rate $R = \frac{K_1}{N_0-(K_0-K_1)}$.



- ◇ Several UEP configurations are reached from the same mother code by changing the preprocess.

Results obtained by pruning a (3,6) LDPC code.





Conclusion

- ◇ We have optimized the check-irregularity of a bit-regular LDPC code to speed up the local convergence of messages, thereby creating UEP behavior.
- ◇ We implemented the cost function by a highly flexible pruning method, that allows to have different UEP configurations with a same mother code.
- ◇ The next step of this work would be to combine bit and check irregularities to provide best unequal error protection.

References

[1] K. Kasai, T. Shibuya, and K. Sakaniwa. Detailedly Represented Irregular Low-Density Parity-Check Codes. *IEICE Trans. Fundamentals*, E86-A(10):2435–2443, October 2003.

[2] S.Y. Chung, T. Richardson, and R. Urbanke. Analysis of Sum-Product Decoding Low-Density Parity-Check Codes using a Gaussian Approximation. *IEEE Trans. on Inform. Theory*, 47(2):657–670, February 2001.